Optimal time scale for spike-time reliability and stochastic synchronization:

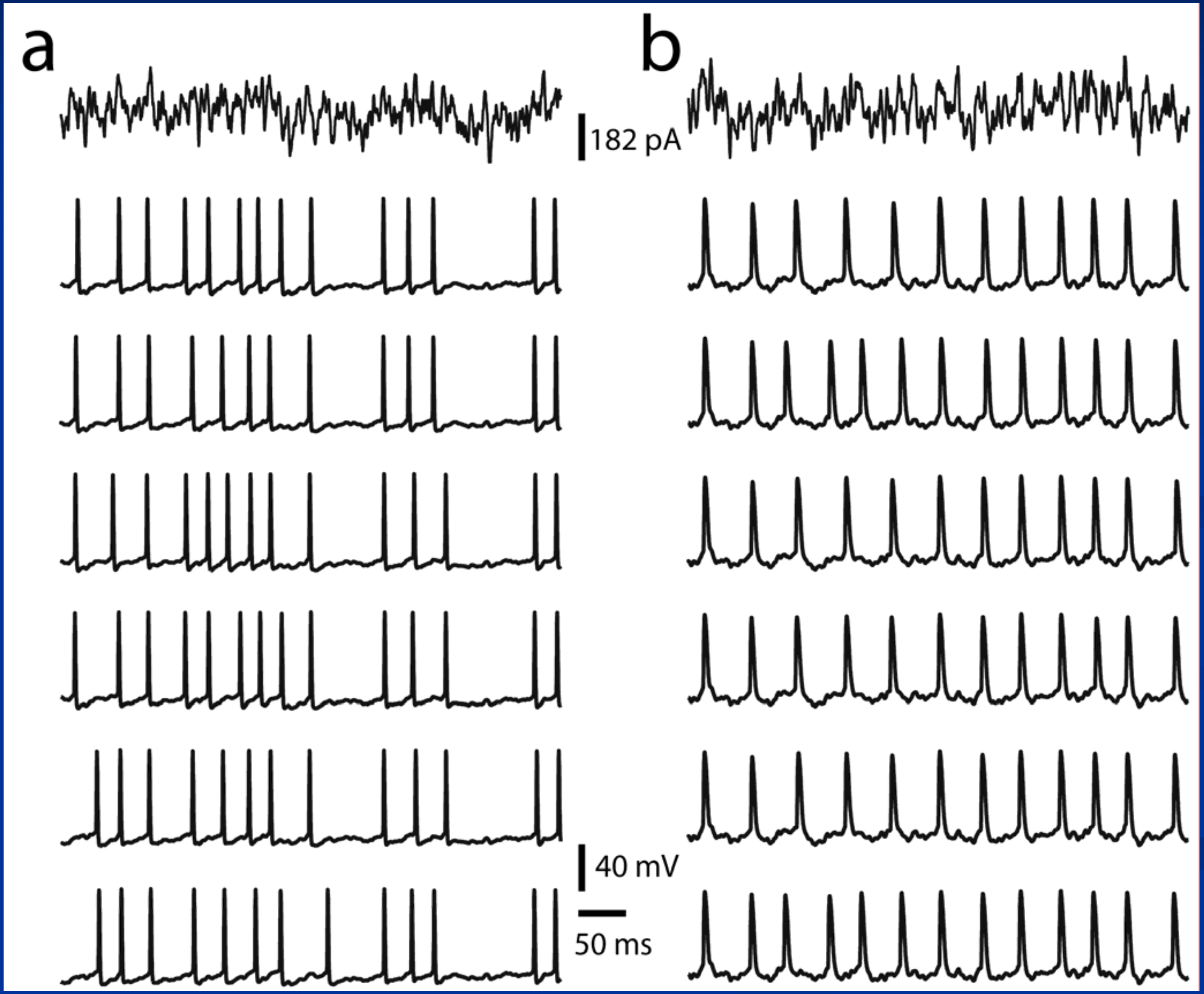
Theory, simulations and experiments

Roberto Fernández Galán,
with
Bard Ermentrout & Nathaniel Urban

Carnegie Mellon University
Pittsburgh Pennsylvania 1900

CNBC
Center for the Neural Basis of Cognition

Spike-time reliability in real neurons



Mitral cell in the olfactory bulb

Pyramidal cell in the neocortex

$x_i(t) = I(t) + \eta_i(t),$

Linear filter approximation of neural dynamics

$$\begin{cases} y_1(t) = \int_0^\infty K(s)x_1(t-s)ds \\ y_2(t) = \int_0^\infty K(s)x_2(t-s)ds \end{cases}$$

output

input

spike-triggered average

Signal + background noise

$$x_i(t) = I(t) + \eta_i(t),$$

fluctuating signal

background noise

$$\langle \eta_1(t)\eta_2(t-s)\rangle = 0, \qquad \langle \eta_i(t)\eta_i(t-s)\rangle = \sigma_\eta^2 \delta(t-s),$$

$$\langle I(t)I(t-s)\rangle = \sigma_I^2 \exp\left(-|s|/\tau\right).$$

Reliability as a cross-correlation

$$R \equiv \frac{\int_{-\infty}^{\infty} y_1(t) y_2(t) dt}{\sqrt{\int_{-\infty}^{\infty} y_1(t) y_1(t) dt \int_{-\infty}^{\infty} y_2(t) y_2(t) dt}} = \frac{\int_{-\infty}^{\infty} y_1(t) y_2(t) dt}{\int_{-\infty}^{\infty} y_1^2(t) dt}$$

Analytical expression of reliability I

$$\int_{-\infty}^{\infty} y_1(t)y_2(t)dt = \int_0^{\infty}\int_0^{\infty} K(s)K(s')\int_{-\infty}^{\infty} x_1(t-s)x_2(t-s')dtdsds' =$$

$$\int_0^{\infty}\int_0^{\infty} K(s)K(s')\langle x_1(t-s)x_2(t-s')\rangle dsds' =$$

$$\sigma_I^2\int_0^{\infty}\int_0^{\infty} K(s)K(s+u)\exp(-u/\tau)dsdu =$$

$$\sigma_I^2\int_0^{\infty}\exp(-u/\tau)\int_0^{\infty} K(s)K(s+u)dsdu =$$

$$Q(u) = \int_0^{\infty} K(s)K(s+u)ds$$

$$\sigma_I^2\int_0^{\infty}\exp(-u/\tau)Q(u)du,$$

Analytical expression of reliability II

$$\int_{-\infty}^{\infty} y_1(t)y_1(t)dt = \int_0^{\infty}\int_0^{\infty} K(s)K(s')\int_{-\infty}^{\infty} x_1(t-s)x_1(t-s')dtdsds' =$$

$$\int_0^{\infty}\int_0^{\infty} K(s)K(s')\left\langle x_1(t-s)x_1(t-s')\right\rangle dsds' =$$

$$\sigma_I^2\int_0^{\infty}\int_0^{\infty} K(s)K(s+u)\exp(-u/\tau)dsdu + \sigma_\eta^2\int_0^{\infty}\int_0^{\infty} K(s)K(s+u)\delta(u)dsdu =$$

$$\sigma_I^2\int_0^{\infty}\exp(-u/\tau)\int_0^{\infty} K(s)K(s+u)dsdu + \sigma_\eta^2 Q(0) =$$

$$Q(u) = \int_0^{\infty} K(s)K(s+u)ds$$

$$\sigma_I^2\int_0^{\infty}\exp(-u/\tau)Q(u)du + \sigma_\eta^2 Q(0).$$

Analytical expression of reliability III

$$R(\tau) = \frac{\sigma_I^2 \int_0^\infty \exp(-u/\tau) Q(u) du}{\sigma_I^2 \int_0^\infty \exp(-u/\tau) Q(u) du + \sigma_\eta^2 Q(0)}$$

$$Q(u) = \int_0^\infty K(s) K(s+u) ds$$

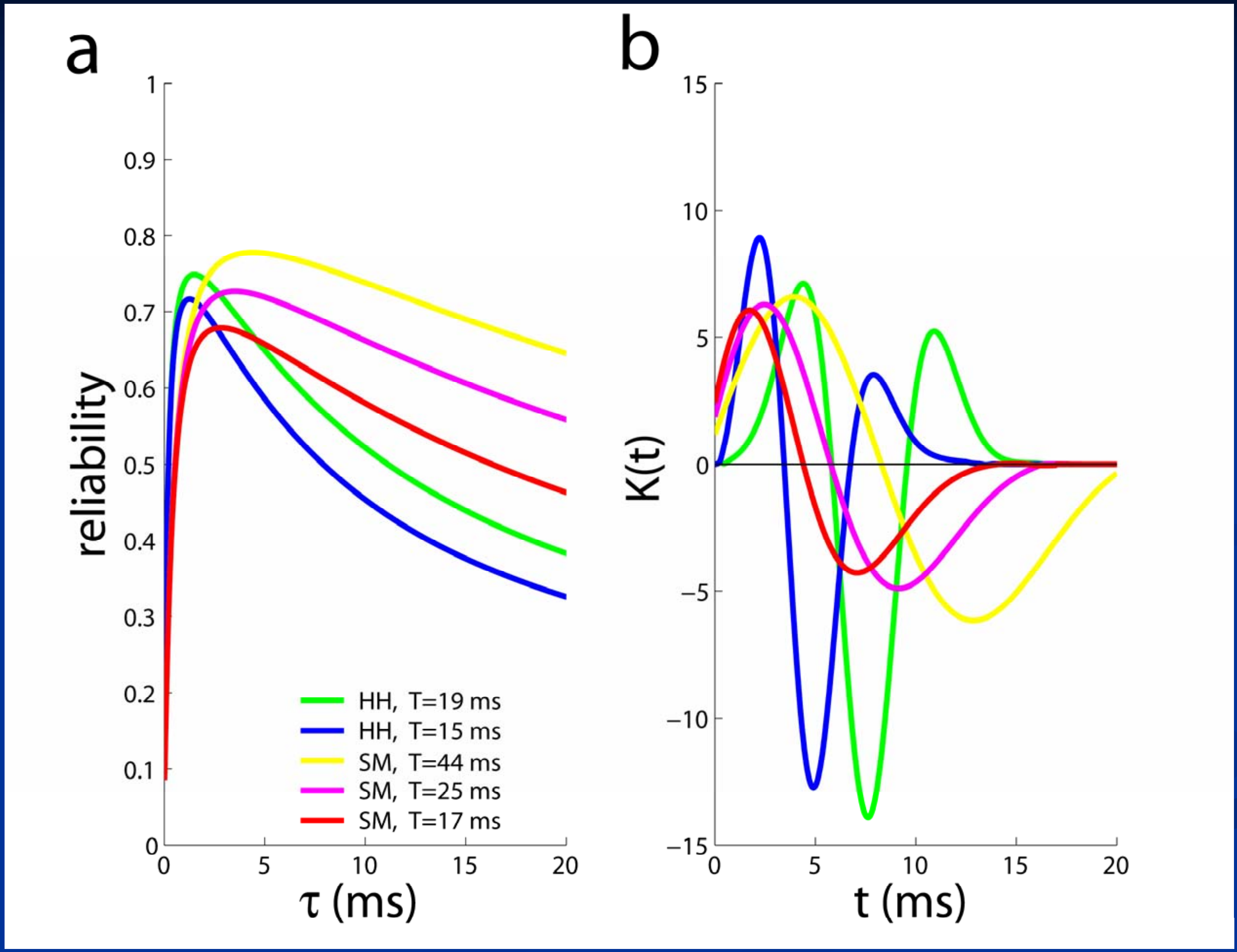
Relationship between the linear kernel and the neuron's phase response

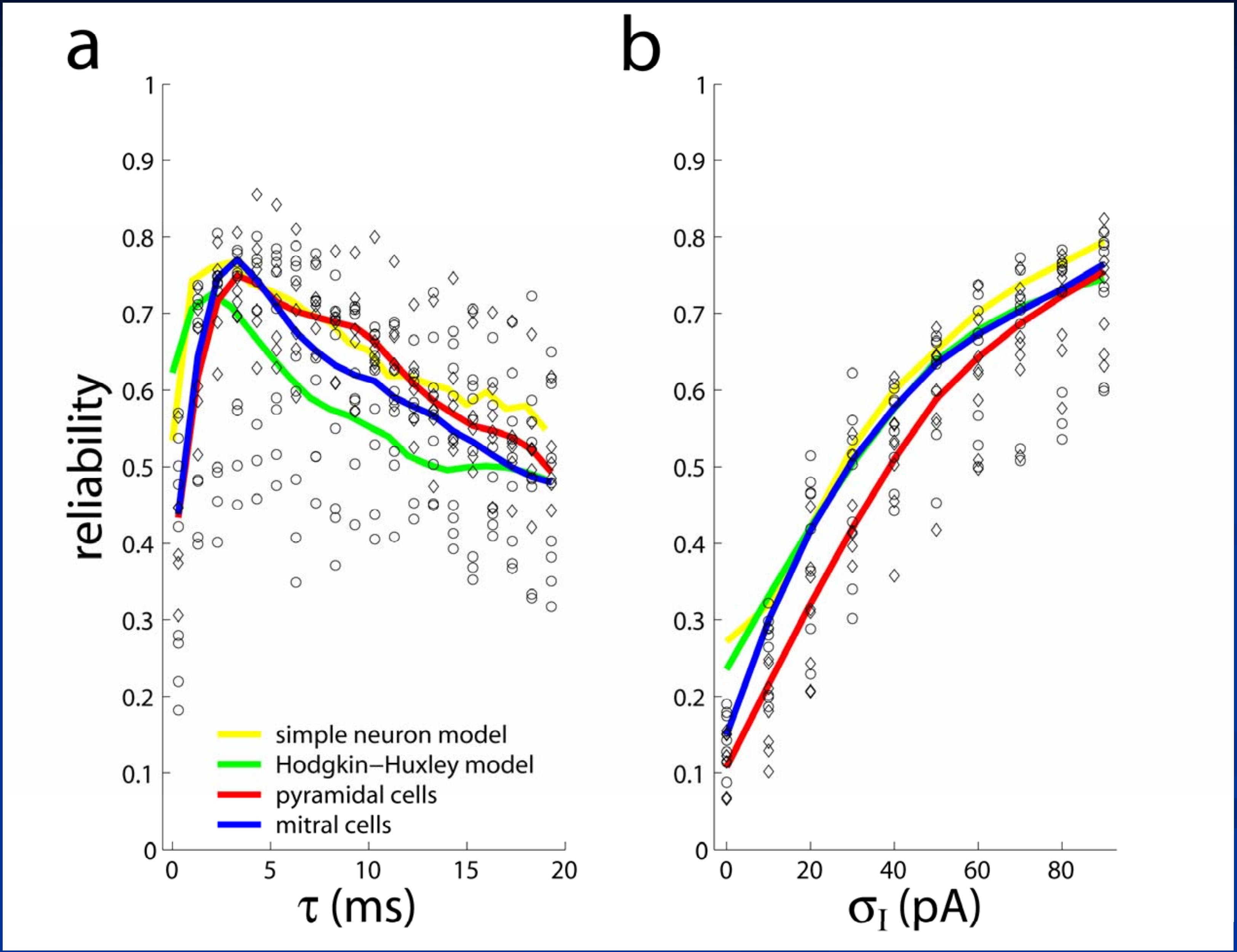
$$PRC(\varphi) = -\int_0^{\varphi} K(t)dt$$

⟺

$$K(t) = -\frac{d}{dt}PRC(t)$$

Ermentrout, Galán and Urban, in prep.


a
b
reliability
τ (ms)
K(t)
t (ms)
HH, T=19 ms
HH, T=15 ms
SM, T=44 ms
SM, T=25 ms
SM, T=17 ms


a
b
reliability
τ (ms)
σI (pA)
simple neuron model
Hodgkin–Huxley model
pyramidal cells
mitral cells

Reliability and Liapunov exponent I

$$\begin{cases} d\theta_1 / dt = 1 + Z(\theta_1)\big(I(t) + \eta_1(t)\big) \\ d\theta_2 / dt = 1 + Z(\theta_2)\big(I(t) + \eta_2(t)\big) \end{cases}$$

$$\phi = \theta_2 - \theta_1$$

$$d\phi / dt = \big(Z(\theta_1 + \phi) - Z(\theta_1)\big) I(t) + \big(Z(\theta_2)\eta_2(t) - Z(\theta_1)\eta_1(t)\big)$$

Reliability and Liapunov exponent II

Using the approximation

$$\left(Z(\theta_1+\phi)-Z(\theta_1)\right)\approx Z'(\theta_1)\phi$$

and averaging in time we obtain an equation of the form

Liapunov exponent

$$d\phi/dt=-\lambda\phi+\sigma_A\eta(t)$$

Reliability and Liapunov exponent III

With

$$\lambda = \lim_{T\to\infty} \frac{1}{T} \int_0^T Z'\big(\theta(t)\big) I(t)\,dt$$

and

$$\sigma_A^2 = \frac{\sigma^2}{\pi} \int_0^{2\pi} Z^2(\theta)\,d\theta$$

Reliability and Liapunov exponent IV

Using the approximation

$$\theta(t) \approx t + \int_0^t Z(s)I(s)ds$$

We obtain

$$\lambda = \lim_{T\to\infty} \frac{1}{T}\int_0^T \left[Z'(t)I(t) + Z''(t)\int_0^t Z(s)I(s)I(t)ds \right] dt \approx$$

$$0 + \lim_{T\to\infty} \frac{1}{T}\int_0^T Z''(t)\int_0^t Z(s)C(t-s)dsdt,$$

Reliability and Liapunov exponent V

$$\lambda \approx \lim_{T\to\infty} \frac{1}{T} \int_0^T Z''(t) \int_0^t Z(s) C(\overbrace{t-s}^{\tau}) ds dt$$

For a sinusoidal phase response, *Z(t)* the absolute value of the Liapunov exponent, λ is maximized close to

$$\tau_{\max} \approx T/(2\pi)$$

Summary

1. Spike-time reliability depends on an intrinsic cell property: the spike-triggered average.

2. In neurons firing periodically, the spike triggered average can be calculated from the phase response curve, which in turn is determined by the equations describing the neuron's dynamics.

3. For neurons firing in the beta/gamma range, spike-time reliability is optimal for signals fluctuating in a time scale of a few milliseconds.

4. This is consistent with the fact that the Liapunov exponent of a phase oscillator with period T is minimal (maximal in absolute value) for fluctuating inputs in the time scale $\sim T/(2\pi)$.

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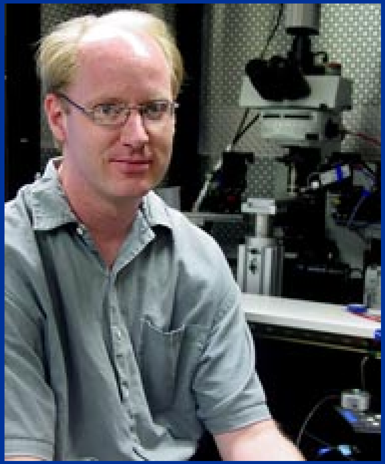

Nathan Urban

Bard Ermentrout