Frequency control in neuronal oscillators using colored noise

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Introduction

It is well-known that the natural frequency of nonlinear oscillators, like neurons, can be strongly affected when driven by a periodic force or when embedded in a network [1]. In contrast, the effects of stochastic forces on the frequency of nonlinear oscillators are incompletely understood. In this context, we have mathematically investigated how colored noise and its amplitude affect the firing frequency of neurons. Our result suggests that low-amplitude colored noise may be used in deep-brain stimulation protocols to control neuronal excitability efficiently.

Methods

The dynamics of a neuron firing spikes (or bursts) regularly can be studied in terms of a phase oscillator model [2], in which the temporal evolution of the phase, φ of the membrane potential between successive spikes is given by:

$$d\varphi / dt = \omega_0 + Z(\varphi) I(t),$$

where ω_0 is the firing frequency (in radians per second) resulting from tonic inputs, i.e. from the DC component of input currents (synaptic or directly injected); $Z(\varphi)$ is the phase-response curve of the neuron, also known as phase sensitivity; and $I(t)$ is a zero-mean, fluctuating component of the input currents. Here we consider $I(t)$ to be colored noise with autocorrelation time, τ, i.e., the autocorrelation function of $I(t)$ decays as:

$$\left\langle I(t) I(t-s) \right\rangle = \sigma^2 \exp\left(-\left| t-s \right| / \tau \right),$$

where σ^2 is the variance of the colored noise, and the brackets denote averaging in time.

Results

We can show mathematically that for a sinusoidal phase-response curve, $Z(\varphi)$ which is a first approximation to the phase response observed in real and simulated neurons [2], the average frequency change due to the colored noise is:

$$\left\langle \Delta\omega \right\rangle \equiv \left\langle d\varphi / dt - \omega_0 \right\rangle = -\frac{\sigma^2}{2} \frac{\omega_0 \tau^2}{1 + \omega_0^2 \tau^2}.$$

Note that the average frequency change is always negative, indicating that the colored noise lowers the natural frequency of the oscillator. In the white-noise limit ($\tau \to 0$), the net frequency change goes to zero. In the limit of slowly varying inputs ($\tau \to \infty$), the net frequency change is $-\sigma^2/(2\omega_0)$. Thus, by combining the amplitude and time constant of the noise, one obtains a wide range for frequency control (Fig. 1). This may have important implications for deep brain stimulation, in particular, to efficiently decrease neuronal excitability in hyperactive areas (e.g., the focus of an epileptic seizure) with minimal current injection.

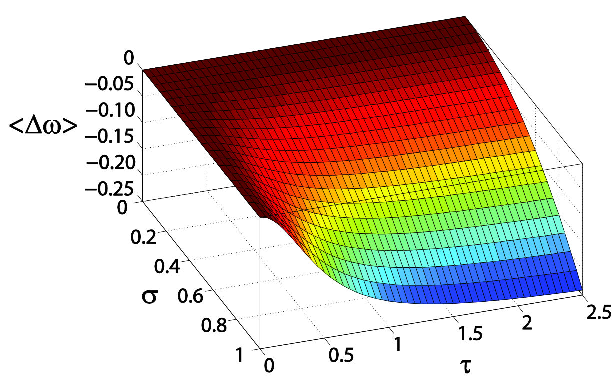

Figure 1: Mean frequency change $\langle \Delta\omega \rangle$, as a function of both, the amplitude, σ and the autocorrelation time, τ of the colored-noise (setting ω_0=2).

Acknowledgements

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References

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