Phase-response curve: predicting network dynamics from a single-cell feature

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Simple neural models that account for…

- …the formation of synchronized cell assemblies, as observed in a variety of neural systems (e.g. Harris, 2003; Plenz & Aertsen, 1996; Wehr & Laurent, 1996).
- …efficient sensory coding at the network level (e.g. Galán et al., 2004; Laurent et al., 2005).
- … stochastic properties of network dynamics, like spike-time reliability (Mainen & Sejnowski, 1994) and noise-induced synchronization (e.g. Teramae & Tanaka, 2004; Galán et al, 2006).

Linear response of neurons

General single-neuron model (e.g. Hodgkin-Huxley)

$$\frac{d\vec{x}}{dt} = F(\vec{x}) + I_0 + I(t)$$

Linearization around a steady state

$$\frac{d\vec{u}}{dt} = L\vec{u} + I(t)$$

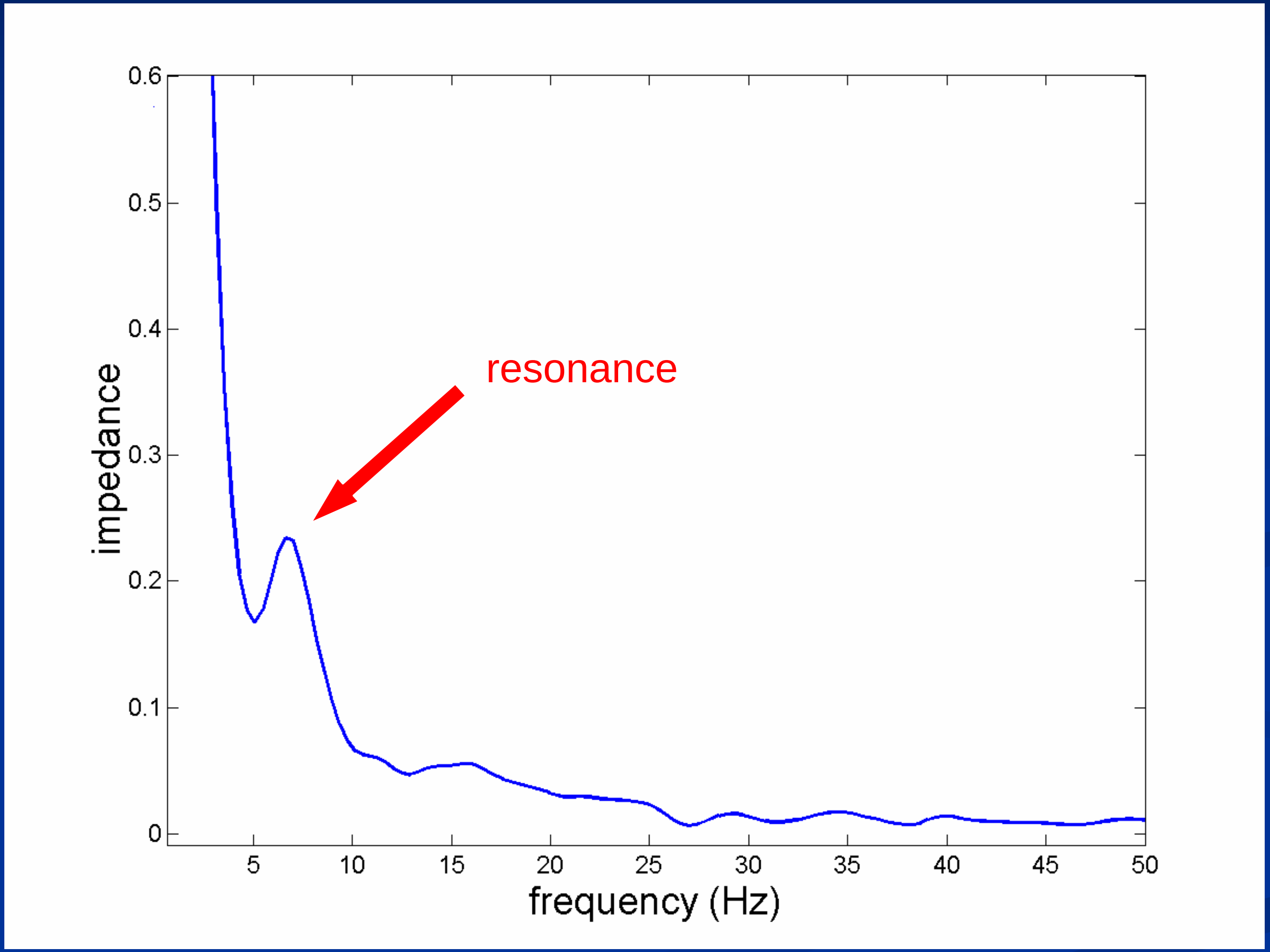
Fourier transform (frequency domain)

$$u_1(\omega) = Z(\omega) I(\omega)$$

$|Z(\omega)|^2$ monotonously decreasing function → Low-pass filter, type I or INTEGRATOR

$|Z(\omega)|^2$ function with a peak → Band-pass filter, type II or RESONATOR

Resonance in mitral cells



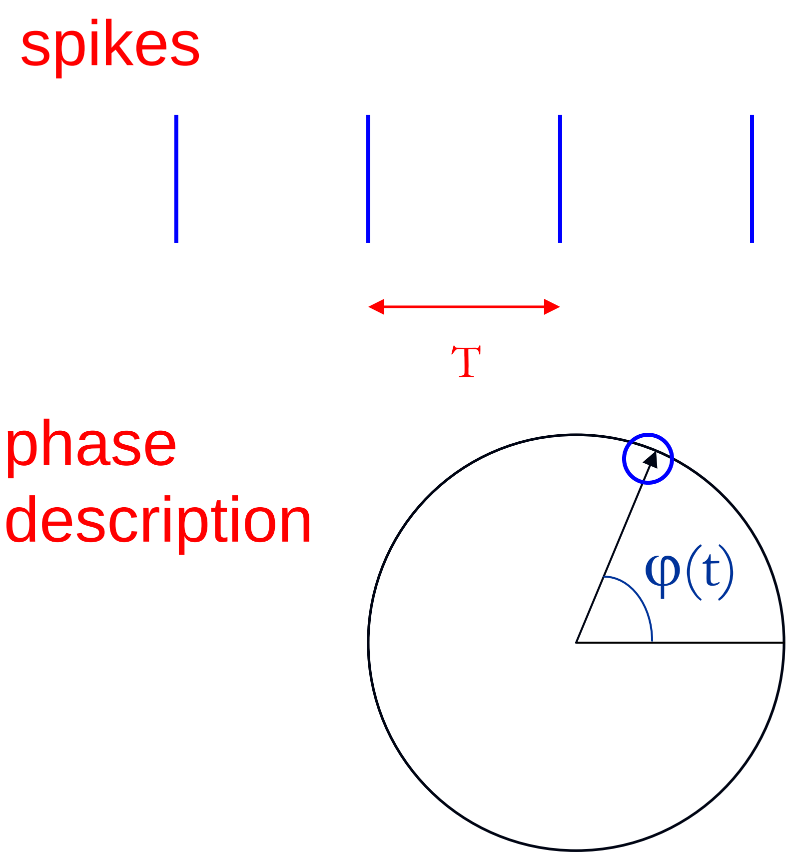
R. F. Galán et al., (in prep.)

Nonlinear response of neurons

- There are several ways of generalizing the concept of impedance to the nonlinear regime of neurons (spiking) in the literature.

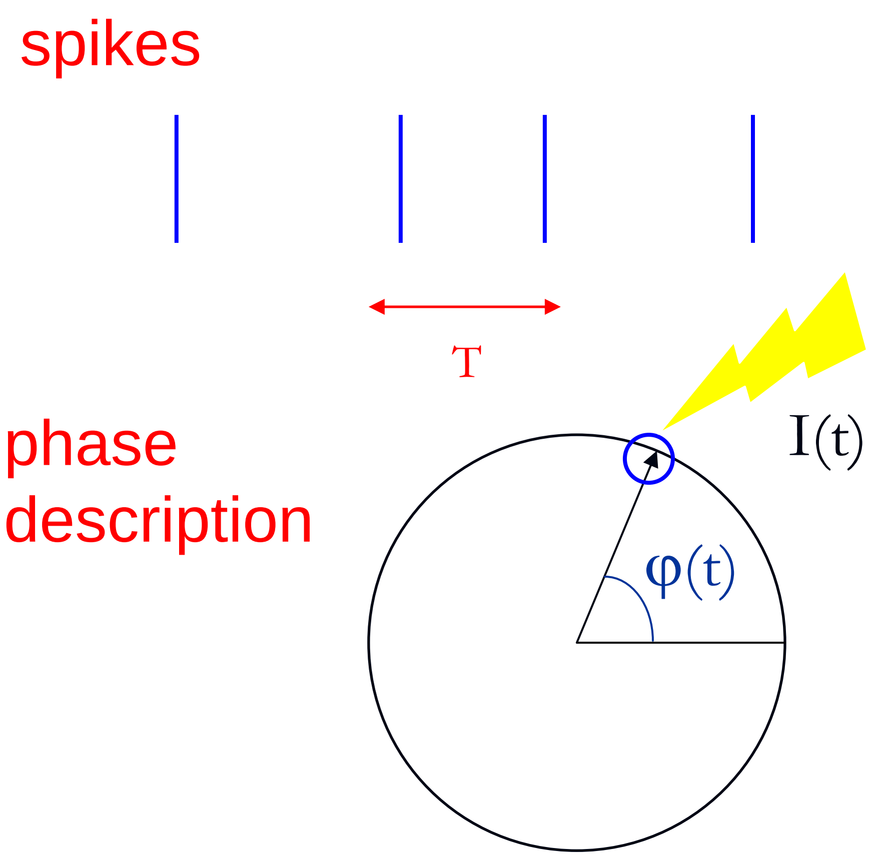
- However, for neurons that fire roughly periodically, the phase-response (or phase-resetting) curve is probably the most intuitive and elegant extension that is also easy to apply to real neurons.

Phase model of a spiking neuron



$$\frac{d\varphi}{dt} = \frac{2\pi}{T} = \omega$$

Phase model of a perturbed neuron

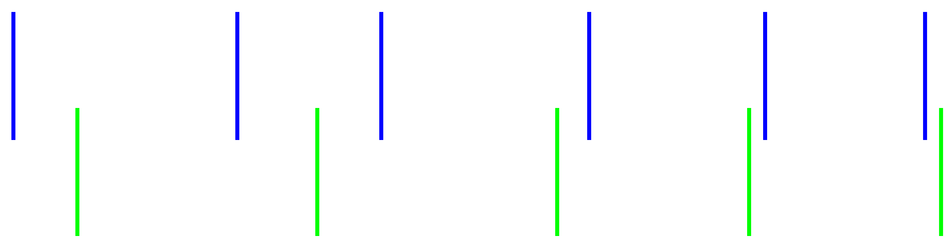


$$\frac{d\varphi}{dt} = \omega + Z(\varphi) \cdot I(t)$$

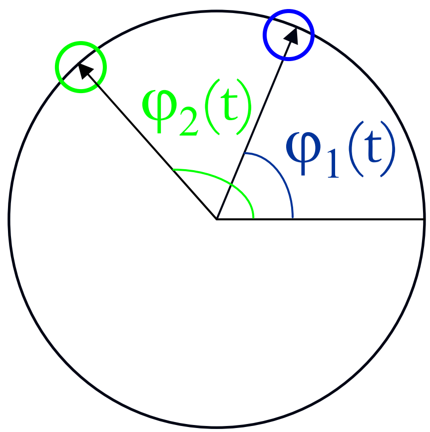
Kuramoto (1986)

Phase model of coupled neurons

spikes



phase description



$$\frac{d\varphi_1}{dt} = \omega_1 + J_{12} \cdot H_1(\varphi_1 - \varphi_2)$$

$$\frac{d\varphi_2}{dt} = \omega_2 + J_{21} \cdot H_2(\varphi_2 - \varphi_1)$$

Network dynamics of coupled phase oscillators

intrinsic frequency

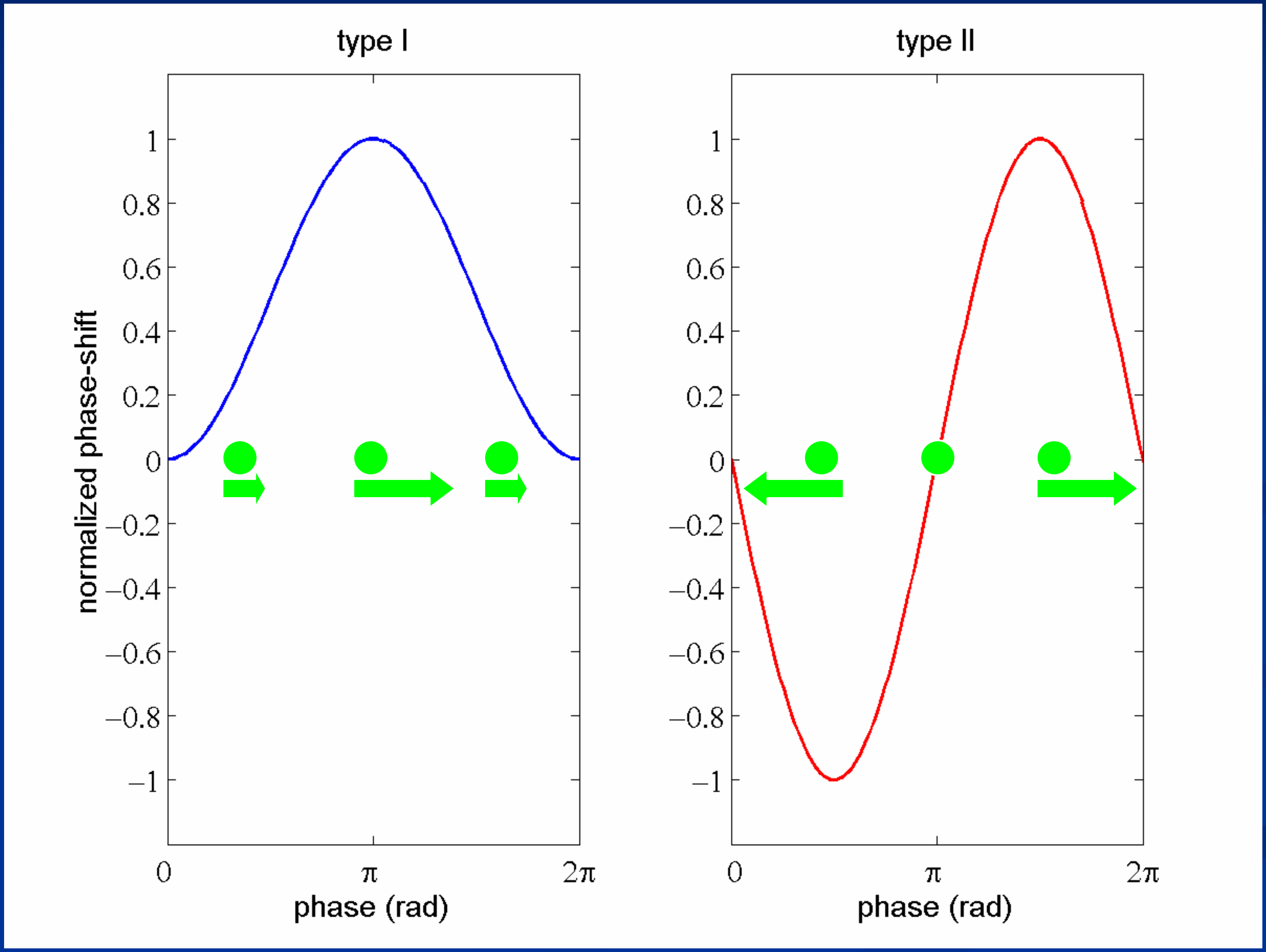
relative phase

$$\frac{d\varphi_i}{dt} = \omega_i + \sum_{j \neq i}^{N} J_{ij} \cdot H_i(\varphi_i - \varphi_j)$$

synaptic sign and strength

interaction function approximated by the **phase-response curve**

Phase-response curves

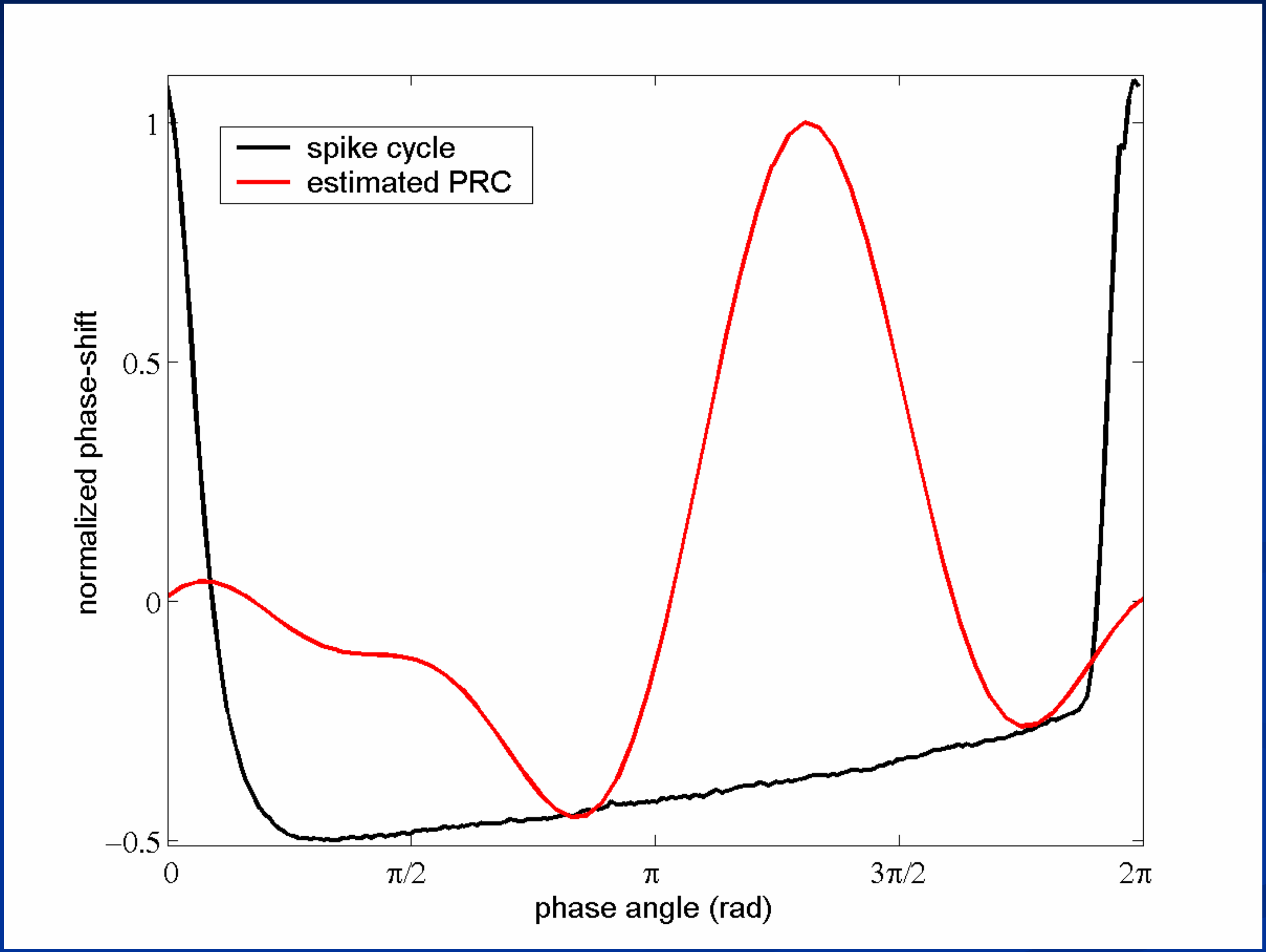


Estimating the PRC of real neurons…

- … by using dynamic clamp techniques.
- … by perturbing the neuron periodically, each time at a different phase of the spike cycle.
- …by using the fact that the PRC is related to the spike-triggered average (Ermentrout et al. in prep.):

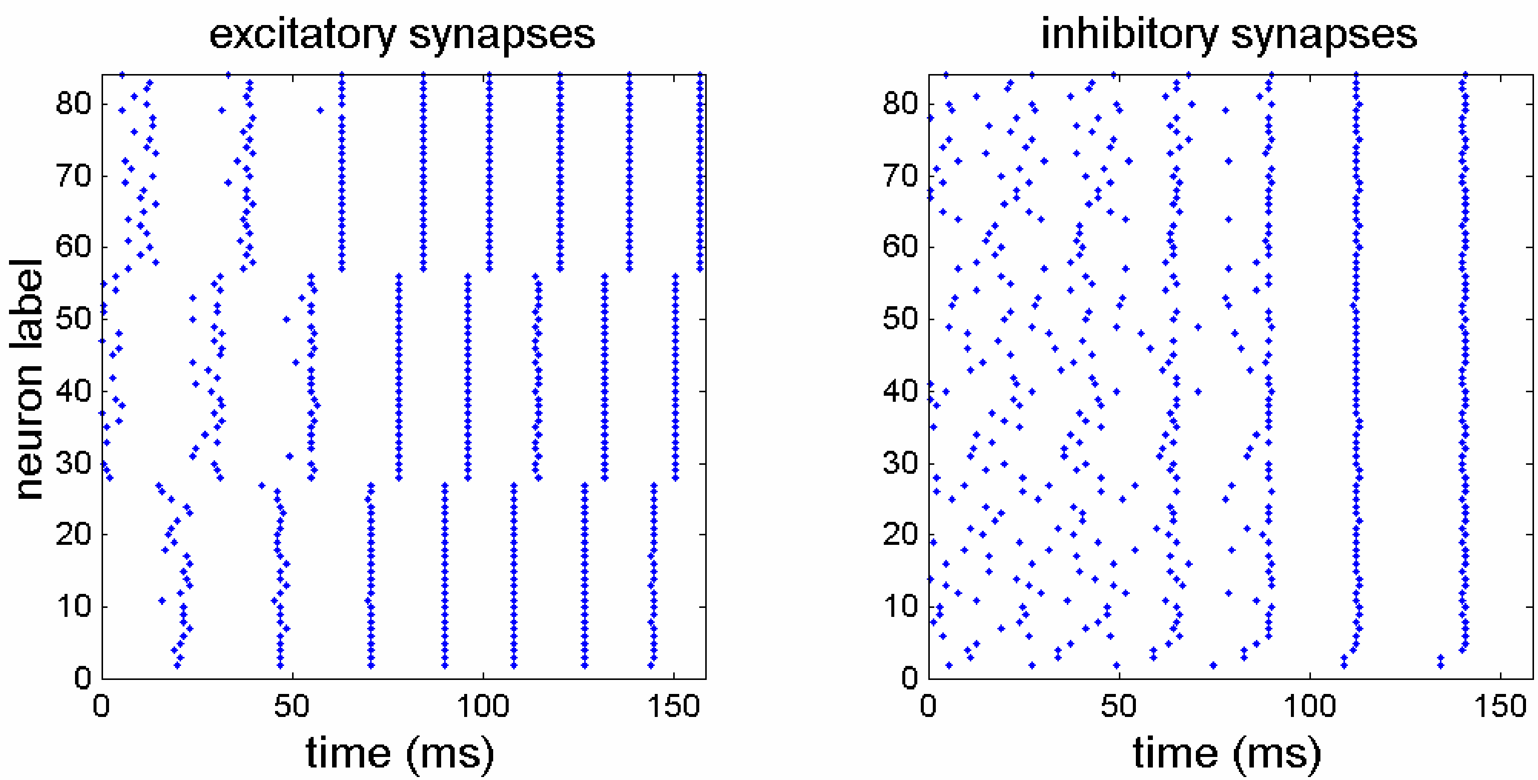
$$PRC(\varphi) = -\int_0^{\varphi} STA(t)\,dt$$

Phase response in mitral cells



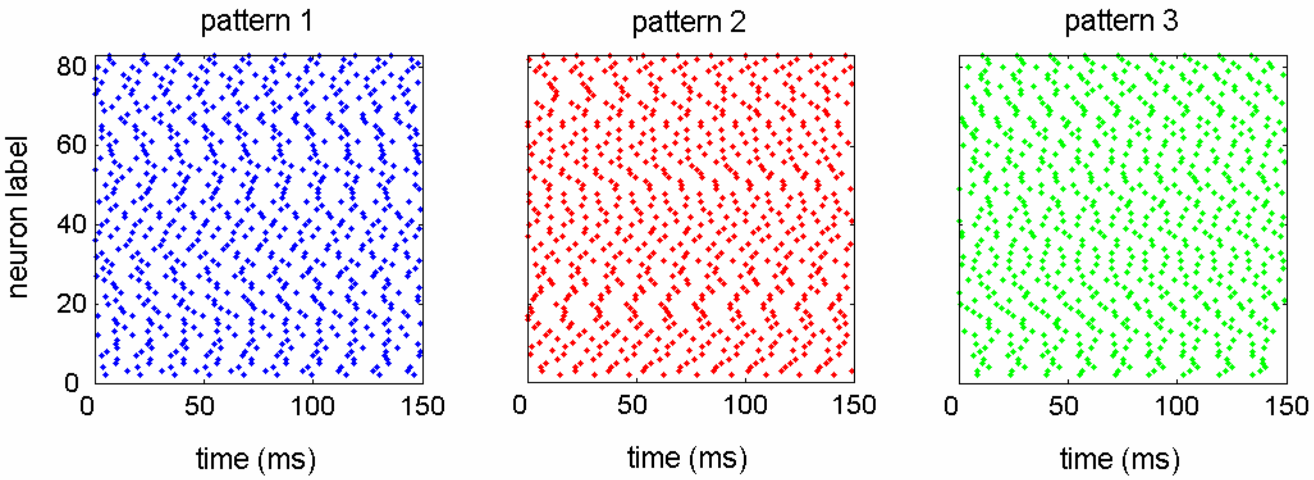
R. F. Galán et al (2005) Phys. Rev. Lett. 94, 158101

Clustering of neural oscillators

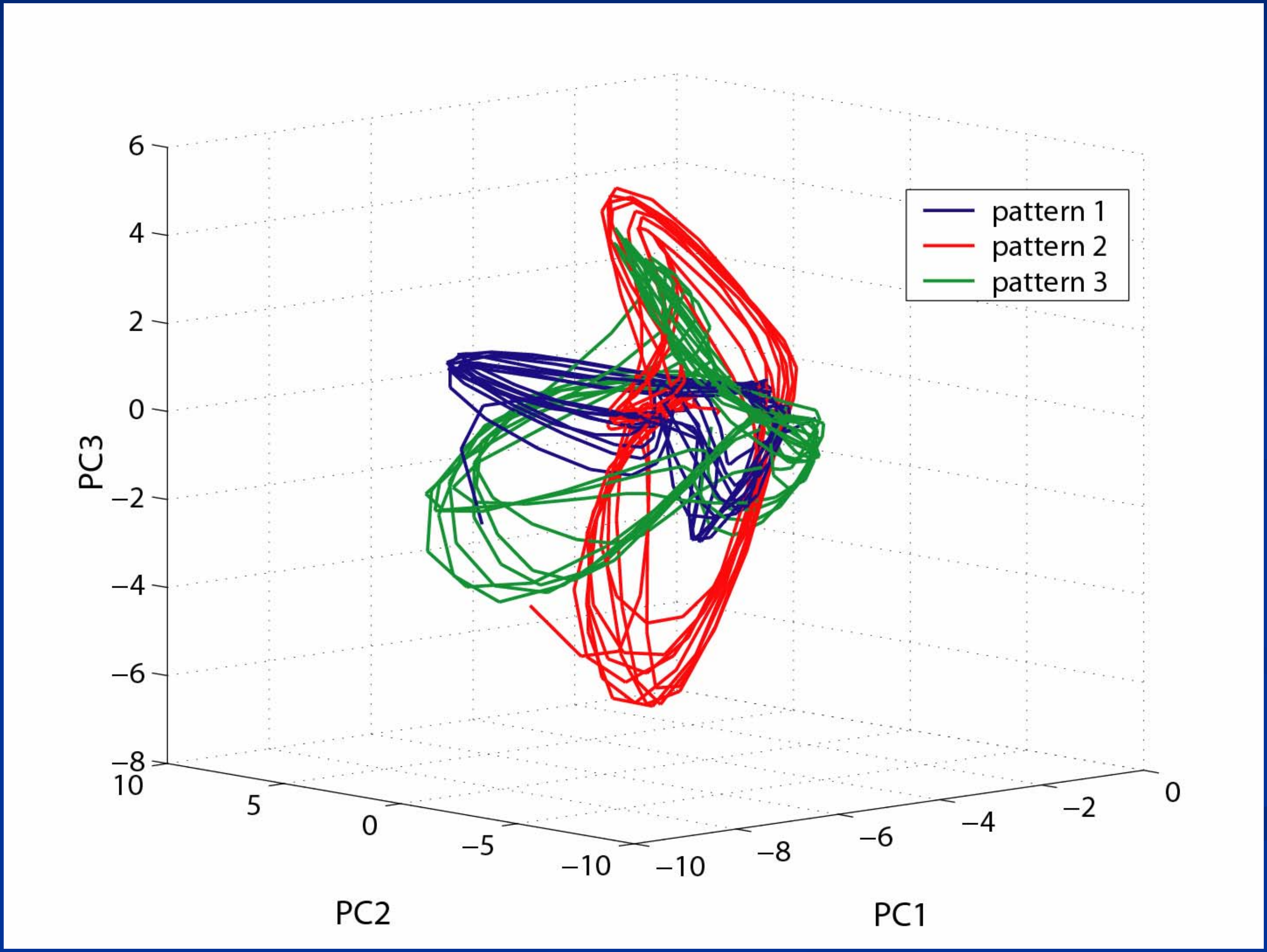


R. F. Galán et al. (2006) Neurocomp. 69(10-12), 1112

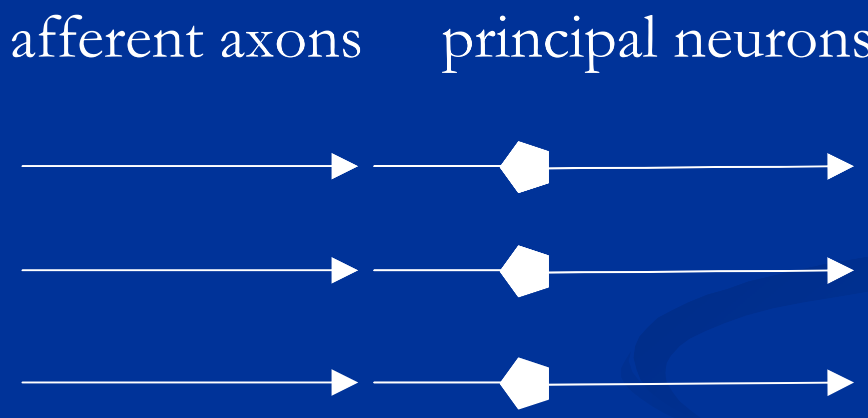
Excitatory and Inhibitory



Embedding of the network dynamics

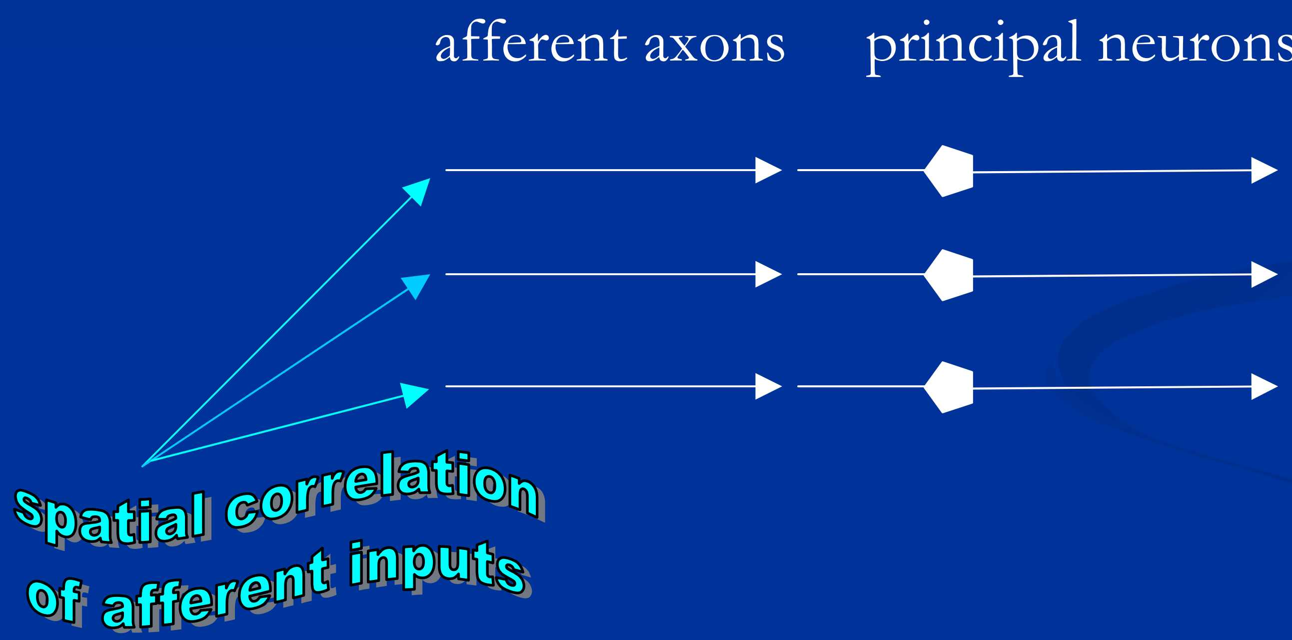


Stochastic synchronization in neuronal networks



R. F. Galán et al. (2006) J. Neurosci. 26, 3646

Stochastic synchronization in neuronal networks



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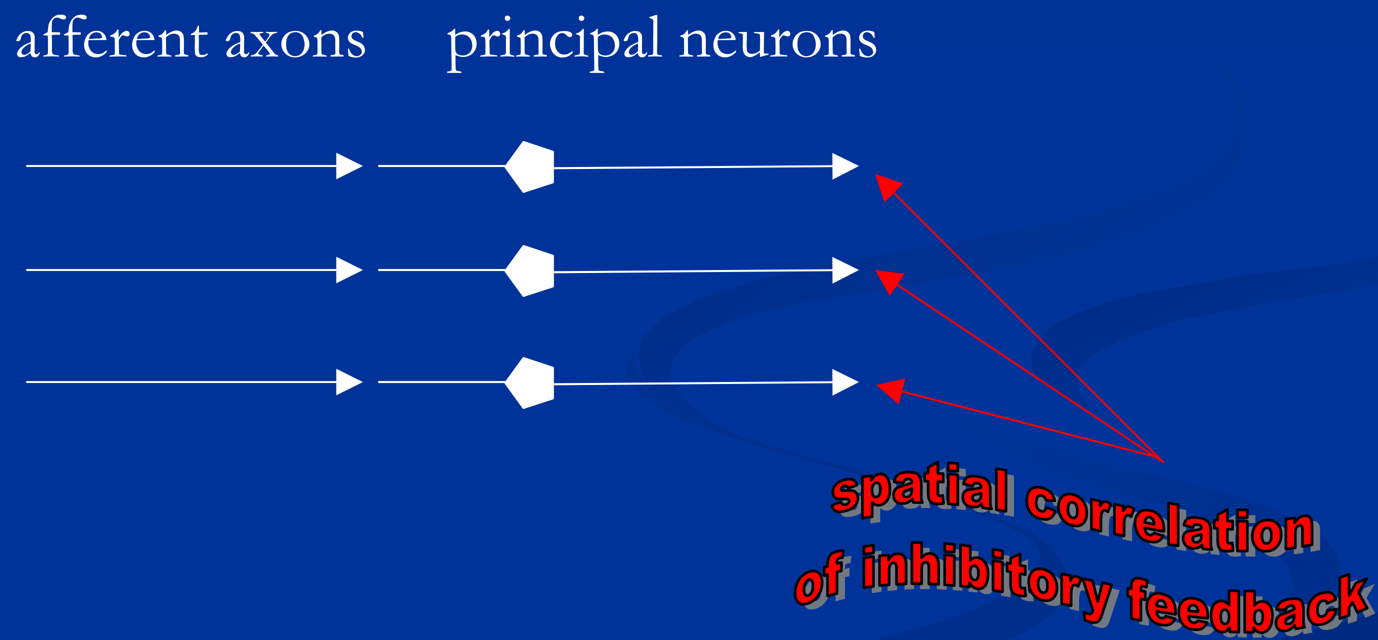

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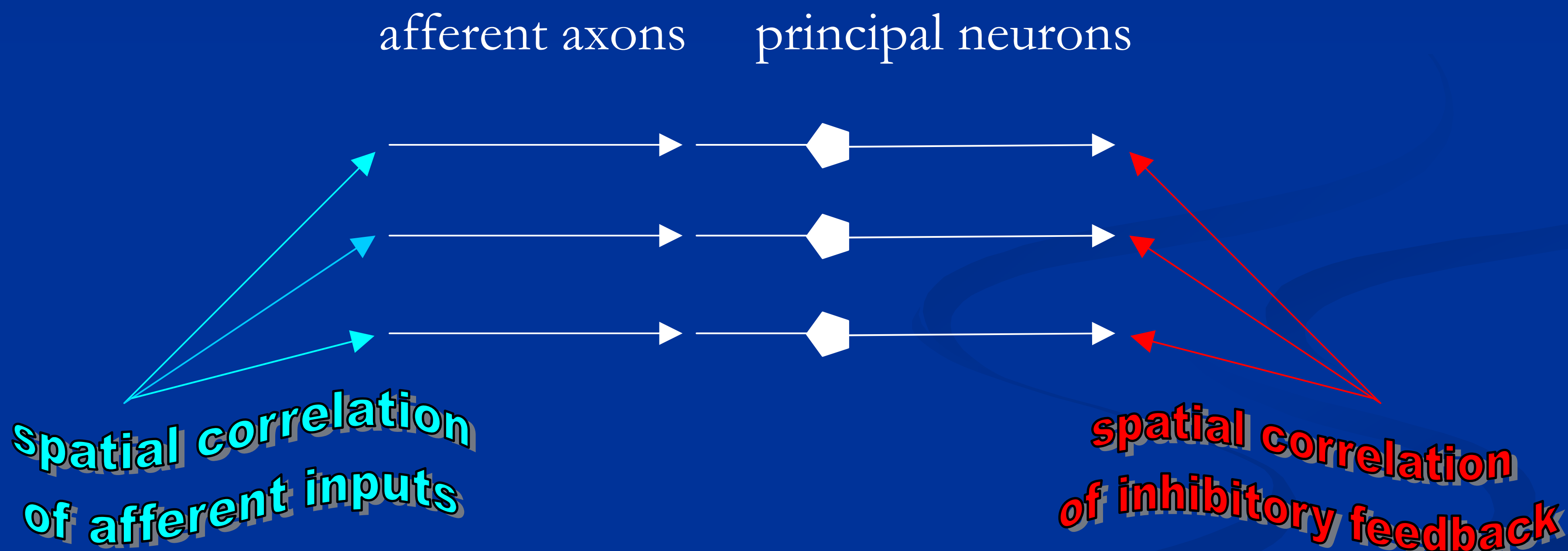
Stochastic synchronization in neuronal networks



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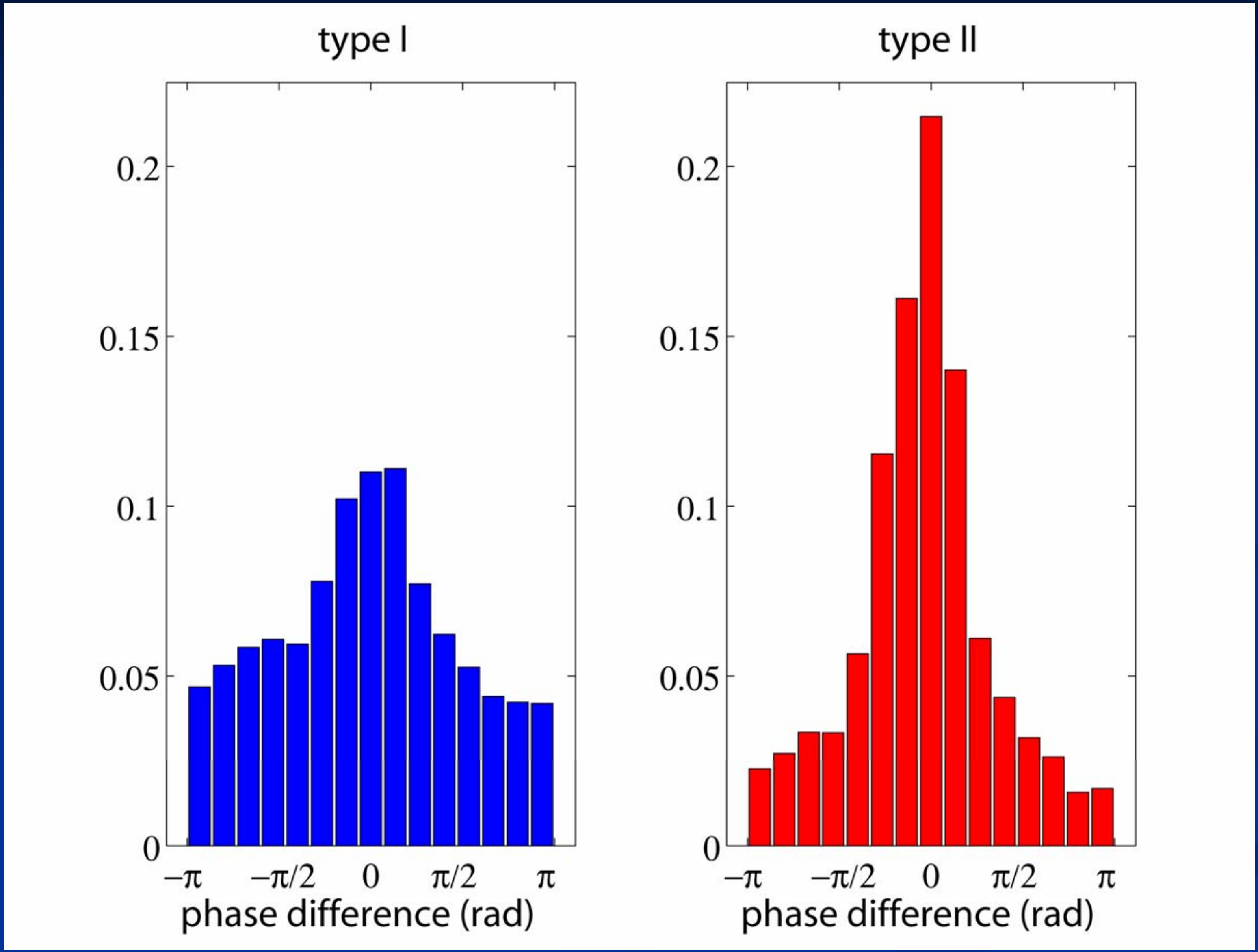
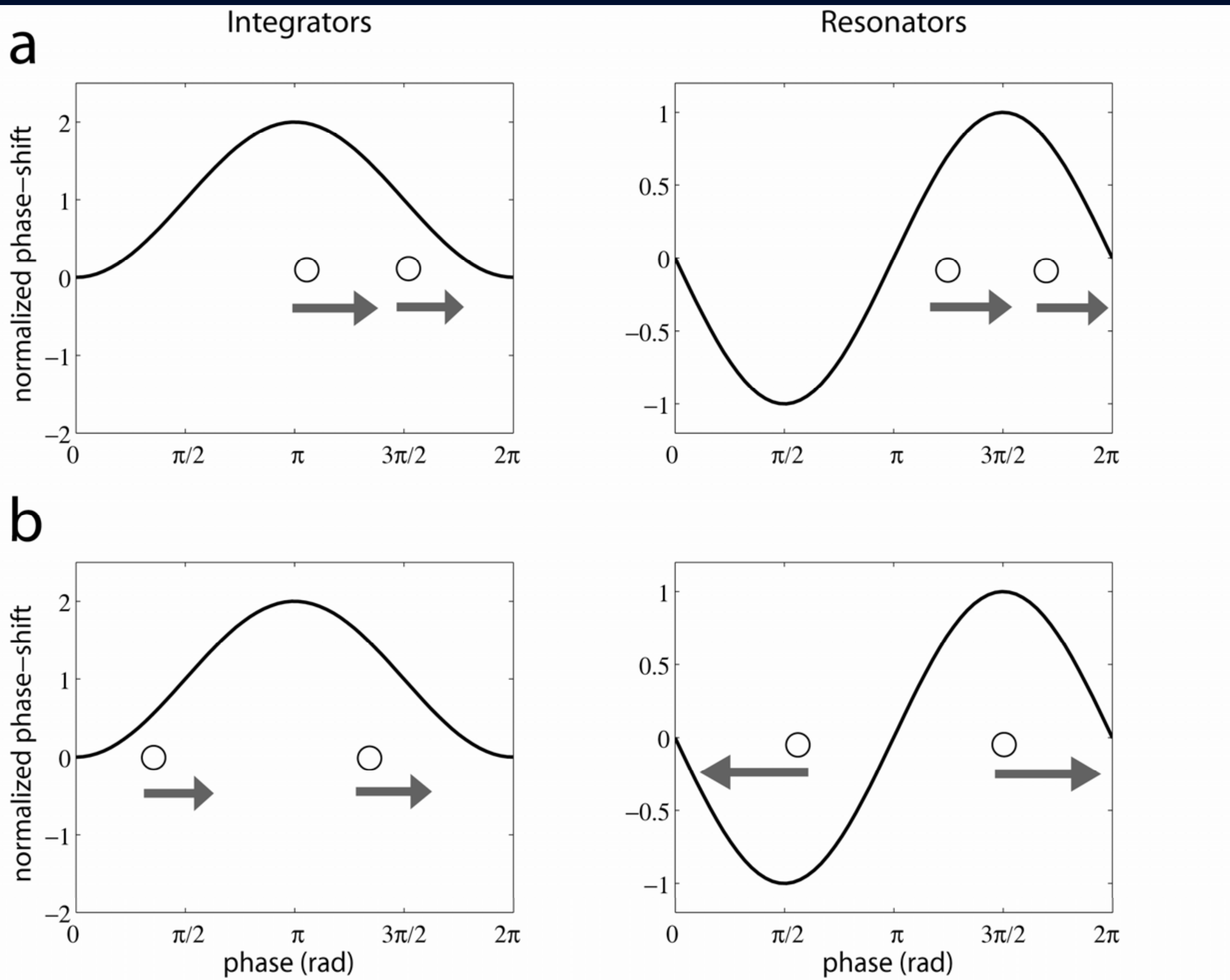
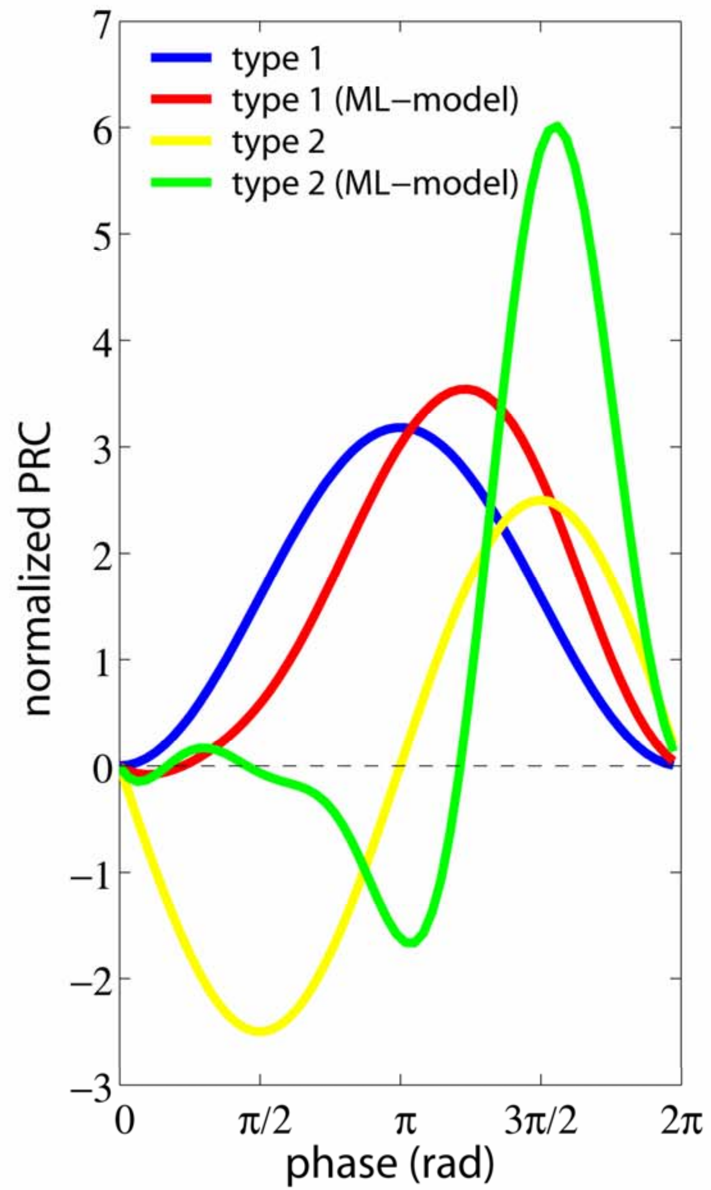
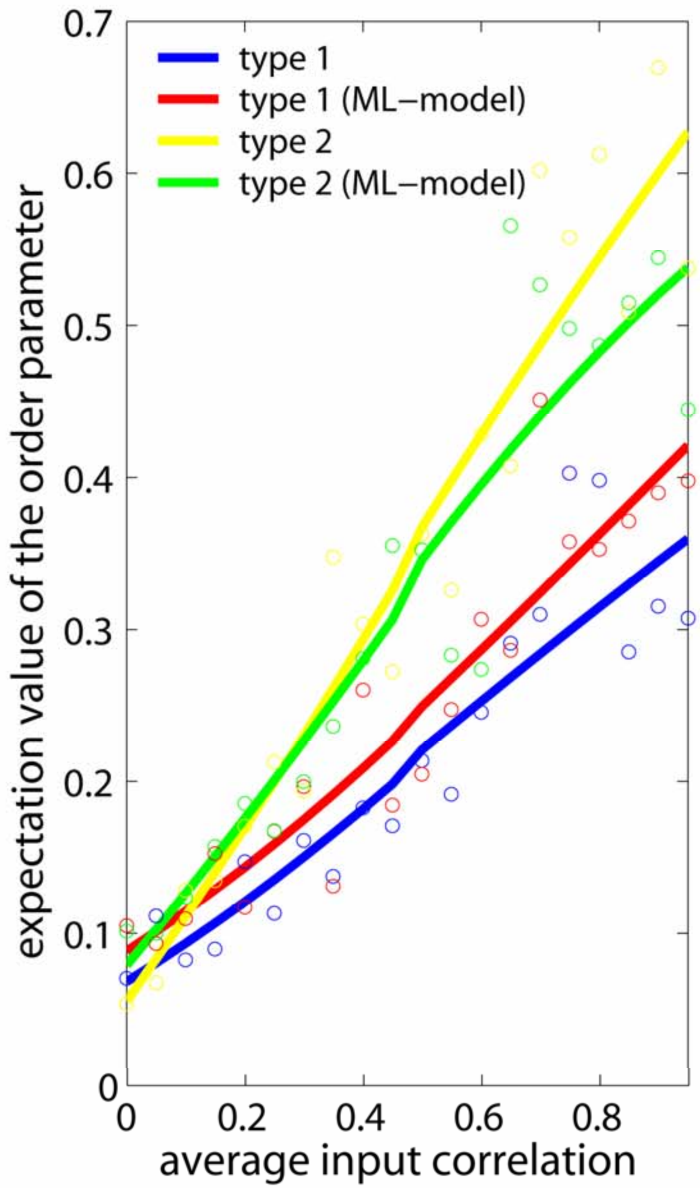

type I
type II
0.2
0.15
0.1
0.05
0
−π
−π/2
0
π/2
π
phase difference (rad)


a
Integrators
Resonators
normalized phase−shift
2
1
0
−1
−2
0.5
−0.5
0
π/2
π
3π/2
2π
b
phase (rad)



type 1
type 1 (ML–model)
type 2
type 2 (ML–model)
normalized PRC
phase (rad)
0
π/2
π
3π/2
2π
7
6
5
4
3
2
1
0
–1
–2
–3



type 1
type 1 (ML–model)
type 2
type 2 (ML–model)
expectation value of the order parameter
average input correlation
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.2
0.4
0.6
0.8

Conclusion

The knowledge of the phase response allows us to construct realistic models (both, deterministic and stochastic) of network dynamics at low computational cost, which remarkably facilitates the study of neural encoding strategies.