Math 402 homework for April 17, 2009

These are problems from chapter 8 of the book by Adkins and Weintraub, edited somewhat.

10: The classification of irreducible complex representations of S_4 in class was incomplete: we did not prove that the two irreducible 3-dimensional representations are inequivalent. Prove this fact as follows.

First recall that the first irreducible 3-dimensional representation was the action of S_4 by permutation of the coordinates on $M = \{(x_1, x_2, x_3, x_4) \in \mathbb{C}^4 \mid \sum x_i = 0\}$. Call this representation α. Then the second irreducible 3-dimensional representation of S_4 was $\alpha' = \alpha \otimes \text{sgn}$.

Show that the characteristic polynomial of $\alpha((1\ 2))$ is $(x-1)^2(x+1)$, but the characteristic polynomial of $\alpha'((1\ 2))$ is $(x-1)(x+1)^2$.

13: Let F and G be arbitrary. If α is an irreducible F-representation of G and β is a 1-dimensional F-representation of G, show that $\alpha \otimes \beta$ is an irreducible representation of G.

21: In *next* week's homework, you will be asked to prove: If G_i is finite and σ_i is an irreducible $\mathbb{C}$-representation of G_i for $i = 1, 2$, then $\sigma_1 \otimes \sigma_2$ is an irreducible representation of $G_1 \times G_2$. (This assignment was originally posted asking you to prove this for arbitrary F, G_1, and G_2; the result is actually false in that level of generality.)

For now, assume the above result is true. If G_1 and G_2 are finite, show that all irreducible $\mathbb{C}$-representations of $G_1 \times G_2$ are of this form.

Hint: use Frobenius's theorem.

30: Let $\sigma : G \to \text{Aut}(V)$ be an irreducible $\mathbb{C}$-representation of G. If $g \in Z(G)$, the center of G, show that $\sigma(g)$ is a scalar transformation of V.