Perspectives in High Dimensions
Schedule

All talks are held in Rockefeller Hall, room 301. Breakfast and registration are outside the lecture hall. Lunches and the banquet are in Hovorka Atrium. The Department of Mathematics is housed in Yost Hall. Yost Hall rooms 300 and 321B are available for discussions.

Monday, August 2

8:15–8:50 Breakfast and registration

8:50–9:00 Welcome by Provost Bud Baeslack

9:00–9:50 Assaf Naor, *Isomorphic uniform convexity in metric spaces*

10:00–10:25 Kevin Ford, *New explicit constructions of RIP matrices and related problems*

10:30–11:00 Coffee, registration

11:00–11:25 Peter Casazza, *Fusion Frames and their Applications*

11:30–11:55 Rick Vitale, *Aspects of Intrinsic Volumes*

12:00–2:00 Lunch break

2:00–2:50 Nathan Linial, *Going up in dimension: Probabilistic and combinatorial aspects of simplicial complexes*

3:00–3:30 Coffee

3:30–3:55 Jaegil Kim, *Local minimality of the volume-product at the simplex*

4:00–4:25 Ronen Eldan, *A Polynomial Number of Random Points does not Determine the Volume of a Convex Body*

Tuesday, August 3

8:15–9:00 Breakfast

9:00–9:50 Marius Junge, *Random subspaces of tensor product and quantum information*

10:00–10:25 Alexander Litvak, *On the rate of convergence of the empirical covariance matrix*

10:30–11:00 Coffee

11:00–11:25 Jiazu Zhou, *On the mixed symmetric homothetic deficit*

11:30–11:55 Beata Randrianantoanina, *Numerical index of some Banach spaces*

12:00–2:00 Lunch break

2:00–2:50 Gideon Schechtman, *Commutators and the structure of isomorphisms on L_p*

3:00–3:30 Coffee

3:30–3:55 Peter Pivovarov, *Mixed volumes of random convex sets*

4:00–4:25 Sanjiv Gupta, *Dichotomy Conjecture on Symmetric Spaces*

4:30–4:55 David Alonso-Gutiérrez, *On a different proof of Junge's estimate for the isotropy constant of polytopes*

Wednesday, August 4

8:15–9:00 Breakfast

9:00–9:50 Emanuel Milman, *Isoperimetric inequalities in semi-convex settings*

10:00–10:25 Paul Goodey, *Local determination and support properties of operators*

10:30–11:00 Coffee and group photo

11:00–11:50 William Johnson, *Dimension reduction in discrete metric geometry*

Thursday, August 5

8:15–9:00 Breakfast

9:00–9:50 Alexander Barvinok, *Maximum entropy formulas for the number of integer points and volumes of polytopes*

10:00–10:25 Pawel Wolff, *On some variants of the Johnson-Lindenstrauss lemma*

10:30–11:00 Coffee

11:00–11:25 Vlad Yaskin, *On unique determination of convex polytopes*

11:30–11:55 Witold Bednorz, *On the compelete characterization of almost sure convergence of all orthogonal series of given coefficients*

12:00–2:00 Lunch break

2:00–2:50 Grigoris Paouris, *Recent progress on the concentration of measure on convex bodies*

3:00–3:30 Coffee

3:30–3:55 Bentuo Zheng, *A nonlinear version of the Grothendieck's theorem*

4:00–4:25 Michael Doré, *Lipschitz functions and uniformly purely unrectifiable sets*

6:00 Banquet

Friday, August 6

8:15–9:00 Breakfast

9:00–9:50 Fedor Nazarov, *On the number of nodal lines of random spherical harmonics*

10:00–10:25 Franz Schuster, *The Symmetry of Bivaluations*

10:30–11:00 Coffee

11:00–11:25 Deping Ye, *On the comparison of volumes of quantum states*

11:30–11:55 Tom Schneider, *70% efficiency of bistate molecular machines explained by information theory, high dimensional geometry and evolutionary convergence*

12:00–2:00 Lunch break

2:00–2:50 Patrick Hayden, *Quantum information theory as high-dimensional geometry*

3:00–3:30 Coffee

3:30–3:55 Łukasz Skowronek, *A symmetry of mapping cones with applications in entanglement theory*

4:00–4:25 Marcin Marciniak, *On exposed positive maps*

Abstracts

David Alonso-Gutiérrez

On a different proof of Junge's estimate for the isotropy constant of polytopes
Tuesday 4:30–4:55 p.m.

Abstract: It was proved by M. Junge that the isotropy constant of any symmetric polytope with $2N$ vertices is bounded by $C \log N$. We give a different proof of this result, which shows that the same estimate is true when the polytope is nonsymmetric with N vertices.

Ahmad Al-Salman

A Class of Marcinkiewicz Integral Operators

Abstract: In recent years the subject of integral operators with rough kernels has undergone a vast development. The interaction of real variable theory with certain geometric properties such as curvature, convexity, and orthogonality is the bright face of this development. The early understanding of the substantial degree of interactions between maximal averages (maximal functions), Fourier transform, and oscillatory integrals has yielded the foundations of a theory of vast scope. In this talk, we are interested in one of the main features of this theory. Namely, we shall introduce a class of Marcinkiewicz integral operators related to Bochner-Riesz operators and Bochner-Riesz summability. We shall discuss the L^p mapping properties of this class of operators when their kernels are rough in $L(\log L)^{1/2}(\mathbf{S}^{n-1})$. The role played by the oscillation carried by the Bessel functions will be highlighted. We shall show that the global parts of the introduced operators are bounded on the Lebesgue spaces L^p $(1 < p < \infty)$ while the local parts are bounded on certain Sobolev spaces. In the last part of the talk, we shall highlight few related operators as well as some possible future research problems.

Alexander Barvinok

Maximum entropy formulas for the number of integer points and volumes of polytopes
Thursday 9:00–9:50 a.m.

Abstract: I plan to discuss computationally efficient "Gaussian" and "almost Gaussian" formulas for the number of integer points and volume of a polytope. The polytope is represented as a section of the non-negative orthant or a cube by an affine subspace. We approximate the counting probability measure, resp. Lebesgue measure, on the section by the maximum entropy distribution on the non-negative integer vectors, resp. non-negative real vectors, with the expectation in the subspace and then use a Local Central Limit argument. Examples include asymptotic formulas for the number of non-negative integer matrices with prescribed row and column sums as well as for the number of graphs with prescribed degrees of vertices. This approach also allows us to obtain concentration results for a random (integer or real) point in a polytope. This is a joint work with John Hartigan (Yale).

Withold Bednorz

On the compelete characterization of almost sure convergence of all orthogonal series of given coefficients

Thursday 11:30–11:55 a.m.

Abstract: In my talk I show a new method of proving the complete characterization of all compact subsets T of real line such that each proces $(X_t)_{t\in T}$ of orthogonal increments is sample bounded. The approach is based on application of weakly majorizing measures and proving a special partition scheme.

Peter G. Casazza

Fusion Frames and their Applications

Monday 11:00–11:25 a.m.

Abstract: Recent advances in hardware technology have enabled the economic production and deployment of a large number of low-cost components, which through collaboration enable reliable and efficient operation. Across different disciplines there is a fundamental shift from centralized information processing to distributed or network-wide information processing. Data communication is shifting from point-to-point communication to packet transport over wide area networks where network management is distributed and the reliability of individual links is less critical. Radar imaging is moving away from single platforms to multiple platforms that co-operate to achieve better performance. Wireless sensor networks are emerging as a new technology with the potential to enable cost-effective and reliable surveillance. All these applications involve a large number of data streams, which need to be integrated at a central processor. Fusion frames are a recent development designed precisely for these applications. Fusion frames allow high dimensional data to be processed in smaller dimensional subspaces with the results later "fused" to produce global outcomes. We will look at fusion frames and some of their applications.

Michael Doré

Lipschitz functions and uniformly purely unrectifiable sets

Thursday 4:00–4:25 p.m.

Abstract: I shall discuss recent progress in the investigation of the differentiability sets of Lipschitz functions defined on Euclidean spaces, and show how some recent results may shed light on a well known conjecture in geometric analysis.

Ronen Eldan

A Polynomial Number of Random Points does not Determine the Volume of a Convex Body

Monday 4:00–4:25 p.m.

Abstract: We show that there is no algorithm which, provided a polynomial number of random points uniformly distributed over a convex body in R^n, can approximate the volume of the body up to a constant factor with high probability.

Kevin Ford

New explicit constructions of RIP matrices and related problems
Monday 10:00–10:25 a.m.

Abstract: We give a new explicit construction of $n \times N$ matrices satisfying the Restricted Isometry Property (RIP). Namely, for some $\varepsilon > 0$, large N and any n satisfying $N^{1-\varepsilon} \le n \le N$, we construct RIP matrices of order $k = n^{1/2+\varepsilon}$ and constant $\delta = n^{-\varepsilon}$. This overcomes the natural barrier $k = O(n^{1/2})$ for proofs based on small coherence, which are used in all previous explicit constructions of RIP matrices. Key ingredients in our proof are new estimates for sumsets in product sets and for exponential sums with the products of sets possessing special additive structure. We also give a construction of sets of n complex numbers whose k-th moments are uniformly small for $1 \le k \le N$ (Turán's power sum problem), which improves upon known explicit constructions when $(\log N)^{1+o(1)} \le n \le (\log N)^{4+o(1)}$. This latter construction produces elementary explicit examples of $n \times N$ matrices that satisfy RIP and whose columns constitute a new spherical code; for those problems the parameters closely match those of existing constructions in the range $(\log N)^{1+o(1)} \le n \le (\log N)^{5/2+o(1)}$. Joint work with Jean Bourgain, Steve Dilworth, Sergei Konyagin and Denka Kutzarova.

Paul Goodey

Local determination and support properties of operators.
Wednesday 10:00–10:25 a.m.

Abstract: About ninety years ago, Blaschke asked whether zonoids are locally determined. Weil's (1977) negative answer to this question led to the question of equatorial support. In recent years, questions such as these have been extended beyond the realm of zonoids. In this talk, I will elaborate a general principle showing that such local or equatorial determination properties are equivalent to corresponding support properties of the associated spherical operators.

Sanjiv Kumar Gupta

Dichotomy Conjecture on Symmetric Spaces
Tuesday 4:00–4:25 p.m.

Abstract: We prove that for any classical, compact, simple, connected Lie group G, the G-invariant orbital measures supported on non-trivial conjugacy classes satisfy a **surprising L^2-singular dichotomy**: For any natural number k, either $\mu_h^k \in L^2(G)$ or μ_h^k is singular to the Haar measure on G. The minimum exponent k for which $\mu_h^k \in L^2$ is specified; it depends on Lie properties of the element $h \in G$. As a corollary, we complete the solution to a classical problem-to determine the minimum exponent k such that $\mu^k \in L^1(G)$ for all central, continuous measures μ on G. Generalisation of this **L^2-singular dichotomy** to symmetric spaces will also be discussed. This is joint work with Prof. K. Hare at the University of Waterloo, Canada.

Patrick Hayden

Quantum information theory as high-dimensional geometry
Friday 2:00–2:50 p.m.

Abstract: Quantum states are represented as vectors in an inner product space. Because the dimension of that state space grows exponentially with the number of its constituents, quantum information theory is in large part the asymptotic theory of finite dimensional inner product spaces. I'll highlight some examples of how abstract mathematical results such Dvoretzky's theorem manifest themselves in quantum information theory as improvements on the famous "teleportation" procedure and reductions in the amount of shared secret information required to encrypt a quantum message, among many other applications.

Alfred Inselberg

Visualizing $\mathbb{R}^N$ and some new dualities

Abstract: With parallel coordinates the perceptual barrier imposed by our 3-dimensional habitation is breached enabling the visualization of multidimensional problems. The highlights, interlaced with interactive demonstrations, are intuitively developed showing how M-dimensional objects are recognized recursively from their $(M-1)$-dimensional subsets. It emerges that *a hyperplane in N-dimensions is represented by $(N-1)$ indexed points.* Points representing lines have two indices, those representing planes in R3 have three indices and so on. In turn, this yields powerful geometrical algorithms (e.g. for intersections, containment, proximities) and applications including classification.

A smooth surface in 3-D is the envelope of its tangent planes each represented by 2 planar points. As a result it is represented by two planar regions, and a hypersurface in N-dimensions by $(N-1)$ regions. This is equivalent to *representing a surface by its normal vectors.* Developable surfaces are represented by curves revealing the surface characteristics. *Convex surfaces in any dimension* are recognized by hyperbola-like regions. Non-orientable surfaces yield stunning patterns unlocking new geometrical insights. Non-convexities like folds, bumps, concavities are visible. The patterns persist in the presence of errors. Intuition gained from the $\mathbb{R}^3$ representations leads to generalizations for $\mathbb{R}^N$ with beautiful new dualities like **cusp in $\mathbb{R}^N \leftrightarrow (N-1)$ swirls in $\mathbb{R}^2$, twist in $\mathbb{R}^N \leftrightarrow (N-1)$ cusps in $\mathbb{R}^2$**. The methodology extends to spaces of dimension $\aleph_0$ and $\aleph_1$.

William B. Johnson

Dimension reduction in discrete metric geometry
Wednesday 11:00–11:50 a.m.

Abstract: In 1981 Lindenstrauss and I proved what has come to be known as the J-L lemma: if A is any set of n points in a Euclidean space, then A can be realized, with constant distortion, in $\mathbb{R}^d$ with $d \leq 1 + \log n$. This means that there is a function F from A into $\mathbb{R}^d$ so that for every pair of points x and y in A,

$|x-y| \le |F(x)-F(y)| \le C|x-y|$. Moreover, F can be taken to be the restriction of a linear mapping from the span of A into $\mathbb{R}^d$ (this stronger version is called the linear J-L lemma). I'll review the proof of this old lemma and mention the application of it in the original paper Joram and I wrote, and then discuss more recent results on dimension reduction, including

1. The theorem of Brinkman and Charikar that the J-L lemma fails in L_1. (The proof I'll outline is an argument due to Schechtman and me that further simplifies the beautiful argument of Lee and Naor.)

2. The result of Naor and mine that while there are spaces other than Hilbert spaces that satisfy the linear J-L lemma, any Banach space that satisfies the lemma must, in a certain sense, be extremely close to a Hilbert space.

Marius Junge

Random subspaces of tensor product and quantum information

Tuesday 9:00-9:50 a.m.

Abstract: Why are random subspaces of tensor products interesting? Dvoretzky's theorem should not care about the tensor product structure. The only reason I can think of are the special applications to quantum information theory. In this talk I will leave the applications to the additivity conjecture aside and show that random subspaces of $l_1(c_0)$, which are Hilbertian up to a logarithmic factor, provide new examples for violation of Bell inequalities. Violation will be explained. This is joint work with Carlos Palazuelos.

Jaegil Kim

Local minimality of the volume-product at the simplex

Monday 3:30–3:55 p.m.

Abstract: It is proved that the simplex is a strict local minimum for the volume-product $P(K) = \min \operatorname{vol}(K) \operatorname{vol}(K^z)$, in the Banach-Mazur space of n-dimensional (classes of) convex bodies. Here K^z is the polar body of K about the point z and the minimum is taken over all the points z in the interior of K. Linear local stability in the neighborhood of the simplex is proved as well. In the proof, methods that were recently introduced by Nazarov, Petrov, Ryabogin and Zvavitch are extended to the non-symmetric setting. This is a joint work with S. Reisner.

Nathan Linial

Going up in dimension: Probabilistic and combinatorial aspects of simplicial complexes

Monday 2:00–2:50 p.m.

Abstract:

The main thesis of my talk is that combinatorics has much to gain by "going up in dimension". We first recall.

Definition 1. *Let V be a finite set of* vertices*. A collection of subsets $X \subseteq 2^V$ is called a* simplicial complex *if it satisfies the following condition: $A \in X$ and $B \subseteq A \Rightarrow B \in X$. A member $A \in X$ is called a* simplex *or a* face *of dimension $|A| - 1$. The dimension of X is the largest dimension of a face in X.*

In theoretical computer science simplicial complexes were used in (i) The study of the evasiveness conjecture, starting with [Kahn, Saks and Sturtevant '83] (ii) Impossibility theorems in distributed asynchronous computation (Starting with [Herlihy, Shavit '93] and [Saks, Zaharoglou '93]).

In combinatorics: (i) Lovász's proof of A. Frank's conjecture on graph connectivity 1977. (ii) Lower bounds on chromatic numbers of Kneser's graphs and hypergraphs. (Starting with [Lovász '78]). (iii) The study of matching in hypergraphs (Starting with [Aharoni Haxell '00]).

The major challenge that we raise is to start a systematic attack on topology from a combinatorial perspective, using the extremal/asymptotic paradigm. In particular we hope to introduce the probabilistic method into topology. In the other direction we suggest to use ideas from topology to develop new probabilistic models (random lifts of graphs offer a small step in this direction). We also hope to introduce ideas from topology into computational complexity.

Can we develop a theory of random complexes, similar to random graph theory? Specifically we seek a higher-dimensional analogue to $G(n, p)$. To fix ideas we consider the simplest possible case: Two-dimensional complexes with a full one-dimensional skeleton. Namely, we start with a complete graph K_n and add each triple (=simplex) independently with probability p. This probability space of two-dimensional complexes is denoted by $X(n, p)$.

We recall from Erdős and Rényi's work:

Theorem 2 (ER '60)**.** *The threshold for graph connectivity in $G(n, p)$ is*

$$p = \frac{\ln n}{n}$$

We next ask when a simplicial complex should be considered connected. Unlike the situation in graphs, this question has many (in fact infinitely many) meaningful answers, i.e.: (i) The vanishing of the first homology (with any ring of coefficients). (ii) Being simply connected (vanishing of the fundamental group).

Theorem 3 (Linial and Meshulam '06)**.** *The threshold for the vanishing of the first homology in $X(n, p)$ over $GF(2)$ is*

$$p = \frac{2 \ln n}{n}$$

This extends to d-dimensional simplicial complexes with a full $(d - 1)$-st dimensional skeleton. Also, for other coefficient groups. (Most of this was done by Meshulam and Wallach). We still do not know, however:

Question 1. *What is the threshold for the vanishing of the homology with integer coefficients?*

I will then report on work Aronshtam, Meshulam and Luczak where we made some progress on the following two questions:

Problem 1. *(i) In this model, what is the threshold for collapsibility?*
(ii) What is the threshold for the vanishing of the second homology?

We next move on to some extremal problems and recall:

Theorem 4 (Brown, Erdős, Sós '73)**.** *Every n-vertex two-dimensional simplicial complex with $\Omega(n^{5/2})$ simplices contains a (triangulation of the) two-sphere. The bound is tight.*

and state:

Conjecture 5. *Every n-vertex two-dimensional simplicial complex with $\Omega(n^{5/2})$ simplices contains a (triangulation of the) torus.*

We can show that if true this bound is tight. This may be substantially harder than the BES theorem, where one actually finds a bi-pyramid. We suspect that such a "local" triangulation of the torus need not exist. With Ehud Friedgut we showed that $\Omega(n^{8/3})$ simplices suffice.

If time permits I will also discuss:

1. Higher-dimensional analogs of permutations and the corresponding enumeration problems.
2. Some recent work with Afek, Feige, Gafni, and Sudakov in the field of distributed computing and how (very elementary) topology comes in.

Alexander Litvak

On the rate of convergence of the empirical covariance matrix.
Tuesday 10:00-10:25 a.m.

Abstract: Let $X_1, ..., X_N \in \mathbb{R}^n$ be an i.i.d. isotropic logconcave random vectors. We discuss how fast $\frac{1}{N}\sum_{i=1}^{N} X_i \otimes X_i$ converges to the covariance matrix $\mathbb{E}X_1 \otimes X_1$.

Marcin Marciniak

On exposed positive maps
Friday 4:00–4:25 p.m.

Abstract: The talk is devoted to the problem of classification of positive maps on C^*-algebras. We consider the class of positive exposed maps. This is a subclass of extremal positive maps. Due to Straszewiczs theorem and its generalizations exposed maps are dense in the set extremal maps. Thus the problem of a full description of positive maps can be reduced to a characterization of all exposed maps. We provide some examples of exposed and non-exposed maps and discuss their properties. In particular we consider the problem of Robertson. He asked if there exists an extremal positive map which satisfies the Schwartz inequality and is not 2-positive. We will show that in the subclass of exposed maps the answer for this problem is negative.

References

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Emanuel Milman

Isoperimetric inequalities in semi-convex settings
Wednesday 9:00–9:50 a.m.

Abstract: In the last two decades, the study of isoperimetric properties of convex domains in high dimensions has become a well established part of the field of Asymptotic Geometric Analysis. This is in part due to a stimulating conjecture of Kannan, Lovász and Simonovits, and to the well known importance of isoperimetric inequalities to the study of Sobolev and concentration inequalities.

We survey some old and recent methods for obtaining isoperimetric inequalities on Riemannian manifolds equipped with a density, under a suitable lower bound on the (generalized) curvature of the space. In particular, we will focus on the Bakry–Émery–Ledoux semi-group method, Gromov's geometric method, isoperimetric profile method, and on contraction methods.

Time permitting, we will describe various applications, ranging from stability of isoperimetric inequalities under perturbation of the underlying measure, to optimal isoperimetric inequalities on compact manifolds with density.

Assaf Naor

Isomorphic uniform convexity in metric spaces
Monday 9–9:50 a.m.

Abstract: Ribe's theorem implies that local isomorphic linear properties of Banach spaces are preserved under uniform homeomorphisms. In the past 30 years there

has been some progress on the problem of finding explicit and quantitative proofs of this general principle for particular properties of Banach spaces. I will survey this research direction and explain some of the known results on non-linear type and cotype. I will then proceed to discuss non-linear versions of the linear Banach space property of having modulus of uniform convexity of power type p. The main theorem is that for Banach spaces, a concrete metric inequality called Markov p-Convexity is equivalent to the classical linear notion of having an equivalent uniformly p-convex norm. Much like the non-linear theory of type and cotype, Markov Convexity is useful for problems in metric geometry which do not necessarily involve linear spaces. But, the metric theory of uniform p-convexity exhibits some surprising differences from the theory of metric type and cotype.

Joint work with Manor Mendel.

Fedor Nazarov

On the number of nodal lines of random spherical harmonics

Friday 9:00–0:50 a.m.

Abstract: We show that the typical number of nodal lines of random spherical harmonics of degree n is proportional to n^2, with exponential concentration around the mean. Many other interesting questions about this number remain wide open.

Grigoris Paouris

Recent progress on the concentration of measure on convex bodies.

Thursday 2:00–2:50 p.m.

Abstract: We will discuss some of the recent developments on various open questions regarding the concentration of measure on convex bodies.

Peter Pivovarov

Mixed volumes of random convex sets

Tuesday 3:30–3:55 p.m.

Abstract: Let $K \subset \mathbb{R}^n$ be a convex body with $\mathrm{vol}(K) = 1$. Let $X_1, \ldots, X_N$ be independent random vectors distributed uniformly in K and form their (symmetric) convex hull $K_N = \mathrm{conv}\{\pm X_1, \ldots, \pm X_N\}$. A result of Groemer's states that the expected volume of K_N,

$$\int_K \cdots \int_K \mathrm{vol}\left(\mathrm{conv}\{\pm x_1, \ldots, \pm x_N\}\right) dx_1 \ldots dx_N$$

is smallest when K is the Euclidean ball B of volume one. A similar result, due to Bourgain, Meyer, Milman and Pajor, holds for the volume of random zonotopes $Z_N = \sum_{i=1}^N X_i$. If $T : \mathbb{R}^N \to \mathbb{R}^n$ is the (random) linear operator defined by $Te_i = X_i$, for $i = 1, \ldots, N$, then K_N is the image of the unit ball in ℓ_1^N, while Z_N is the image of the unit ball in ℓ_∞^N. What happens when T is applied to other sets? I will discuss a unified approach to various inequalities involving mixed volumes of random convex

sets for which the Euclidean ball is the minimizer. Joint work with Grigoris Paouris, Texas A & M University.

Beata Randrianantoanina

Numerical index of some Banach spaces
Tuesday 11:30–11:55 a.m.

Abstract: Let X be a Banach space and $\Pi(X) := \{(x, x^*) \in S_X \times S_{X^*} \ : \ x^*(x) = 1\}$. For an operator $T \in L(X)$, its *numerical radius* is defined as

$$v(T) := \sup\{|x^*(Tx)| \ : \ (x, x^*) \in \Pi(X)\},$$

which is a seminorm on $L(X)$ smaller than the operator norm. The *numerical index* of X is the constant given by

$$n(X) := \inf\{v(T) \ : \ T \in L(X),\ \|T\| = 1\} = \max\{k \geq 0 \ : \ k\,\|T\| \leq v(T)\ \forall T \in L(X)\}.$$

The numerical radius of bounded linear operators on Banach spaces was introduced, independently, by F. Bauer and G. Lumer in the 1960's extending the Hilbert space case from the 1910's.

To date numerical radius of very few spaces is evaluated exactly. In this talk I will present recent joint results with J. Merí, M. Martín and M. Popov concerning numerical index of absolute sums of Banach spaces and some estimates for the values of numerical index in Lebesgue and Orlicz spaces.

Gideon Schechtman

Commutators and the structure of isomorphisms on L_p
Tuesday 2:00–2:50 p.m.

Abstract: This will be an announcement of two related results obtained recently by Dosev, Johnson and the speaker. The first is a characterization of the (bounded, linear operators which are) commutators on L_p spaces, i.e. the operators on L_p which are of the form $AB - BA$, as exactly those operators which cannot be written as $aI + S$ with non zero a, and S which is "L_p strictly singular" namely not an isomorphism when restricted to any subspace isomorphic to L_p. Except for tools previously used in characterization of commutators on simpler spaces, the main new tool is a theorem on the structure of isomorphisms on L_p spaces which strengthen results from a work of Johnson, Maurey, Tzafriri and the speaker. In particular, for any isomorphism $T : L_p \to L_p$ there is a subspace X of L_p such that on X and TX there are bounded linear projections from L_p; the constants involved (including the isomorphism constant of $T_{|X}$!) do not depend on the original isomorphism but only on p. Moreover, for $1 < p < 2$, the assumption that T is an isomorphism can be weaken to T being a "sign embedding".

Tom Schneider

70% efficiency of bistate molecular machines explained by information theory, high dimensional geometry and evolutionary convergence

Friday 11:30–11:55 a.m.

Abstract: The relationship between information and energy is key to understanding biological systems. We can display the information in DNA sequences specifically bound by proteins by using sequence logos, and we can measure the corresponding binding energy. These can be compared by noting that one of the forms of the second law of thermodynamics defines the minimum energy dissipation required to gain one bit of information. Under the isothermal conditions that molecular machines function this is Emin = kB T ln 2 joules per bit (kB is Boltzmann's constant and T is the absolute temperature). Then an efficiency of binding can be computed by dividing the information in a logo by the free energy of binding after it has been converted to bits. The isothermal efficiencies of not only genetic control systems, but also visual pigments are near 70%. From information and coding theory, the theoretical efficiency limit for bistate molecular machines is ln2 = 0.6931. Evolutionary convergence to maximum efficiency is limited by the constraint that molecular states must be distinct from each other. The result indicates that natural molecular machines operate close to their information processing maximum (the channel capacity), and implies that nanotechnology can attain this goal.

Franz Schuster

The Symmetry of Bivaluations

Friday 10:00–10:25 a.m.

Abstract: Many powerful isoperimetric and related inequalities involve fundamental operators on convex bodies which are valuations, e.g., centroid, projection, and intersection body maps. In many instances the proofs of these inequalities are based on the symmetry of certain *bivaluations* associated with convex body valued valuations. These symmetry properties are in turn deeply intertwined with the self adjointness of well known integral transforms such as Radon and cosine transforms and, consequently, played a critical role in the solution of a number of problems. In this talk I want to present a recent joint work with S. Alesker and A. Bernig in which we established the symmetry of homogeneous bivaluations compatible with rigid motions in full generality, extending several previous partial results. As applications of these symmetry properties, I will present new Brunn–Minkowski and Busemann–Petty type inequalities. (Joint work with Semyon Alesker and Andreas Bernig.)

Łukasz Skowronek

A symmetry of mapping cones with applications in entanglement theory

Friday 3:30–3:55 p.m.

Abstract: Mapping cones, introduced by Størmer in [1], are a special class of linear maps on operator algebras, including k-positive maps. They appear in a

number of problems in quantum information science. Under the weak assumption that they are closed under conjugation, a surprisingly strong characterization theorem holds, saying that a map is an element of a mapping cone $\mathcal{K}$ iff its product with any element of the cone dual to $\mathcal{K}$ is completely positive. The result builds elegantly on the definition of the Hilbert-Schmidt inner product and the fact that the Jamiołkowski-Choi isomorphism is an isometry. We discuss this and give examples how the characterization theorem for mapping cones can be applied to concrete problems. In particular, we rederive in an instructive way a result by Chruściński & Kossakowski [3] on k-positive with a single negative eigenvalue in their Choi matrix. We also discuss norms defined relative to a mapping cone (cf. [2]) and give a sufficient criterion for the second tensor power of a map to be 2-positive. The last problem is directly related to two-copy distillability of quantum states [4]. Joint work with Erling Størmer.

References

[1] E. Størmer, Extension of positive maps into $B(H)$, *J. Funct. Anal.* **66**, 235 (1986)

[2] N. Johnston, D. Kribs, Schmidt norms for quantum states, preprint arXiv:0909.3907v2

[3] D. Chruściński, A. Kossakowski, Spectral Conditions for Positive Maps, *Commun. Math. Phys.* **290**, 10511064 (2009)

[4] D. DiVicenzo et al., Evidence for bound entangled states with negative partial transpose, *Phys. Rev. A* **61**, 062312 (2000)

Sasha Sodin

Random band matrices: eigenvalue statistics and quantum dynamics

Abstract: I will try to survey some problems regarding the distribution of eigenvalues of random band matrices, and describe some recent progress based on resummation of a series expansion, in particular:

- the distribution of eigenvalues close to the edge of the spectrum
- the results of L. Erdos and A. Knowles on the quantum dynamics on short time scales
- estimates on the density of states

Rick Vitale

Aspects of Intrinsic Volumes

Monday 11:30–11:55 a.m.

Abstract: As key functionals in the Brunn-Minkowski theory of convex bodies, intrinsic volumes enter into a number of different questions. We will discuss some of these instances together with open problems, as time permits.

Pawel Wolff

On some variants of the Johnson-Lindenstrauss lemma
Thursday 10:00–10:25 a.m.

Abstract: The Johnson-Lindenstrauss lemma states that a set of n points in Euclidean space R^d can be mapped to R^k with $k = O(logn)$ so that all distances between the points are nearly preserved. In a typical proof one considers a random operator from R^d to R^k (e.g. a random orthogonal projection or Gaussian random matrix) and shows that with positive probability such operator works for a given set of n points.

I will recall two recent approaches to the Johnson-Lindenstrauss lemma, which are computationally more efficient than standard ones - one due to Ailon and Chazelle based on sparse random matrices and the other, due to Vybiral, based on partial circulant random matrices. Some improvements, especially addressing the amount of randomness used in these constructions, will be presented.

Vlad Yaskin

On unique determination of convex polytopes.
Thursday 11:00–11:25 a.m.

Abstract: We will discuss the following two open problems. 1) A question of Barker and Larman asks whether convex bodies that contain a sphere of radius t in their interiors are uniquely determined by the volumes of sections by hyperplanes tangent to the sphere. 2) In his book "Geometric Tomography" Gardner asks whether origin-symmetric convex bodies in $\mathbb{R}^3$ are uniquely determined by the perimeters of sections through the origin. One can also formulate an n-dimensional version of the problem.

We solve these problems in the class of convex polytopes.

Deping Ye

On the comparison of volumes of quantum states
Friday 11:00–11:25 a.m.

Abstract: Entangled (i.e., not separable) quantum states play fundamental roles in quantum information theory; therefore, it is important to know the "size" of entanglement (and hence separability) for various measures, such as, Hilbert-Schmidt measure, Bures measure, induced measure, and α-measure.

In this talk, I will present new comparison results of α-measure with Bures measure and Hilbert-Schmidt measure. Employing these comparison results to the subsets of separable states and of states with positive partial transpose, we show that the probability of separability is very small, and the well-known Peres-Horodecki PPT Criterion as a tool to detect separability is imprecise for (even moderate) large dimension of Hilbert space.

This talk is based on my papers: J. Math. Phys. 50 (2009) 083502, and J. Phys. A: Math. Theor. in press.

Jiazu Zhou

On the mixed symmetric homothetic deficit
Tuesday 11:00–11:25 a.m.

Abstract: In this talk, we are going to introduce the mixed symmetric homothetic deficit of the convex sets $K_1, \cdots, K_n$ in the Euclidean space R^n. The isohomothetic inequality and some Bonnesen-style isohomothetic inequalities are obtained. We will report our recent works on the lower and upper limits of the mixed symmetric homothetic deficit. Finally, we will discuss the Gage isoperimetric inequality, Ros isoperimetric inequality and more Gage-type and Ros-type inequalities.

Bentuo Zheng

A nonlinear version of the Grothendieck's theorem
Thursday 3:30–3:55 p.m.

Abstract: We use a recent factorization theorem of Farmer and Johnson for Lipschitz p-summing operators to show that every Lipschitz mapping f from a metric tree to a Hilbert space is Lipschitz 1-summing hence Lipschitz 2-summing. Moreover, the Lipschitz 2-summing norm of f is bounded by A_1^{-1} multiplying the Lipschitz constant of f, where A_1 is the Khinchine's constant.