# Math. 449: Dynamical models for Biology and Medicine

**D. Gurarie**

Yost 104. M/W: 2-3:15pm

**Class Objectives:**

- Introduce basic concepts and ideas from several fields of Biology, Medicine, Physiology (population dynamics, ecology, genetics, bio-chemistry, immunology, spread of infectious diseases)
- Develop mathematical and computational methods, tools and techniques of dynamical systems in the context of Bio-medical models and applications

**Course Description:** This course is intended for graduate and upper level undergraduate students in Applied Math., Biology, Medicine, Sciences and Engineering. It introduces some basic mathematical ideas, techniques and tools for modeling Bio-Medical systems and processes. The models range from population dynamics of single and interacting species, to bio-chemistry (enzyme kinetics, metabolism, biological feedback- control), basic immunology and infectious disease transmission.

The mathematical exposition is accompanied and supplemented by computational methods and tools, based on Wolfram Mathematica and Matlab.

The course covers a broad range of dynamical systems, continuous and discrete. But no advanced knowledge of differential equations or dynamical systems is required, beyond the standard calculus sequence 224-228, and linear algebra 201. We shall review some basic topics in discrete and continuous dynamics: equilibria, stability, bifurcations, linear and nonlinear oscillations, qualitative analysis, partial differential equations, pattern formation), in the context of Bio-Medical applications.

**Prerequisites**:

- Math 224/228 or BIOL/EBME 300. Recommended prerequisites: Math 234, or Math 338, Math 201 (or equivalent),
- consent of instructor.

**Course work** will consist of lectures, based on the textbook, and additional sources (notes, books, papers).

**Grading**

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| i) assigned homework: | - 30% |
| ii) class attendance/ presentation | -20% |
| iii) exam | -20% |
| iii) final project | -30% |

**Text**: **N.F. Britton**, Essential Mathematical Biology, Springer, 2003

**Additional sources:**

**J. D. Murray**, Mathematical Biology, Springer, 2002
**L. Edelstein-Keshet**, Mathematical models in Biology, SIAM, 2005
**R.M. Anderson**, R.M. May, Infectious diseases of humans, Oxford UP, 1998;
**M. Novak and R. May** "Viral dynamics: the mathematical foundations of virology and immunology", by, Oxford UP, 2000

**Software:** Mathematica, Matlab (available via CWRU software center).

## Tentative Syllabus

1. Single population dynamics (Ch. 1)
    a. Discrete and continuous population models
    b. Metapopulations
    c. Age structure
2. Population dynamics of interacting species (Ch.2)
    a. Host-parasitoid interactions
    b. The Volterra - Lotka predator-prey: equilibria, stability, phase-plane analysis
    c. Competition
3. Infectious diseases (Ch. 3)
    a. Population models (SEIR methodology): persistence, eradication and control
    b. Vector-borne diseases
    c. Macro-parasite diseases
    d. Individual-based models
4. Population genetics and evolution (Ch.4)
    a. Mendelian genetics and selection
    b. The balance between selection and mutation
    c. Evolution of genetic systems
5. Biological motion (Ch. 5)
    a. Macroscopic theory of motion and Chemotaxis
    b. Biological invasion
    c. Traveling wave solutions of reaction-diffusion equations
6. Molecular and cellular biology (Ch. 6)
    a. Bio-chemical kinetics and metabolic pathways
    b. Neural modeling
    c. Basic immunology and HIV/AIDS
7. Pattern formation
    a. Turing instability and bifurcations in activator-inhibitor systems
    b. Incorporating biological movement; mechano-chemical models
8. Tumor modeling

**Bulletin description**

Introduction to discrete and continuous dynamical models with applications to Biology and Medicine. Topics include: population dynamics and ecology; models of infectious diseases; population genetics and evolution; biological motion (reaction-diffusion and chemotaxis); Molecular and cellular biology (biochemical kinetics, metabolic pathways, immunology). The course will introduce students to the basic mathematical concepts and techniques of dynamical systems theory (equilibria, stability, bifurcations, discrete and continuous dynamics, diffusion and wave propagation, elements of system theory and control). Mathematical exposition is supplemented with introduction to computer tools and techniques (Mathematica, Matlab).
Prerequisites: Math 224/228 or BIOL/EBME 300, Math. 201 or equivalent