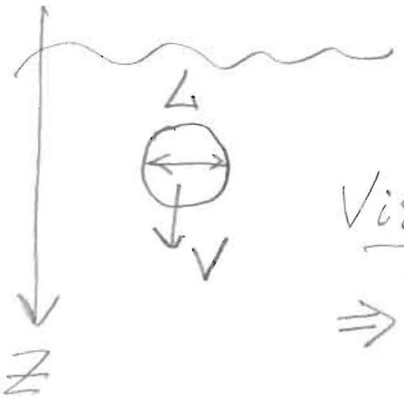
D. GURARIE

B-dary source: diffusion w. sedimentation

Plankton



Length velocity

Reynolds #: $\boxed{Re = \frac{LV}{\nu_K}}$ kinematic viscosity

Viscous friction: $F_{fr} = \alpha V$; $\boxed{\alpha = \frac{c_0 \, mass}{Re}}$

⇒ "small particle" ⇒ $Re \ll 1 \Rightarrow \alpha \gg 1$

Vertical motion: $V' = -\alpha V + g \Rightarrow V(t) = \boxed{\frac{g}{\alpha}} + ce^{-\alpha t}$

Limiting velocity

$u(z,t)$ – plankton mass density

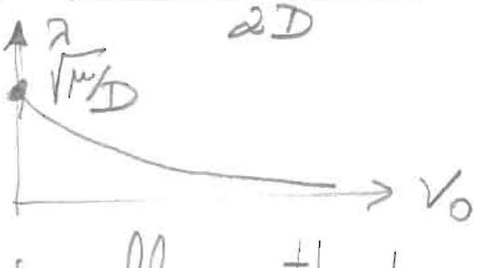
$$u_t = -\underbrace{V_0 u_x}_{\text{sediment.}} + \underbrace{D u_{xx}}_{\text{diff}} - \underbrace{\mu u}_{\text{mortality}}; \quad z > 0$$

BC: $u|_{z=0} = B(t)$

Equilibrium solution (B - const)

$$u_0(z) = Be^{-\lambda z}; \quad \lambda = \frac{\sqrt{V_0^2 + 4\mu D} - V_0}{2D}$$

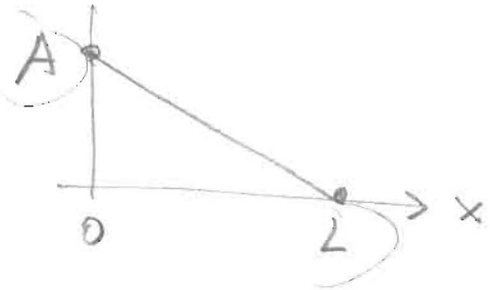
Small particles ($Re \ll 1$, $V_0 \ll 1$) spread deeper



Exercise: Show numerically that $u(z,t) \to u_0(z)$ as $t \to \infty$

Transit time

Equil. profile: $\begin{cases} Du_{xx} = 0 & \text{on } [0, L] \\ u|_0 = A; \; u|_L = 0 \end{cases}$



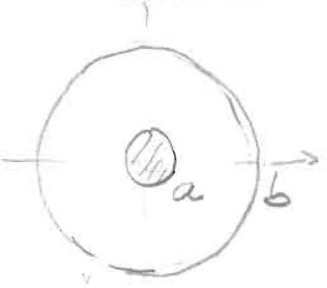
$$\Rightarrow \boxed{u(x) = A\left(1 - \frac{x}{L}\right)}$$

Particle #: $N = \int_0^L u(x,t)\,dx = \frac{AL}{2}$

Current $= \frac{\#\text{ part.}}{\text{time}} = \text{flux} = \frac{DA}{L}$

$$\boxed{\text{Time} = \frac{N}{J} = \frac{L^2}{2D}}$$

Disk case (5.11): $\begin{cases} \Delta u = 0; & a < r < b \\ u|_a = A; \; u|_b = 0; & 0 < \theta < 2\pi \end{cases}$



Radial solution: $u(r)$

$$\begin{cases} u_{rr} + \frac{1}{r}u_r = 0 \\ u|_a = 1; \; u|_b = 0 \end{cases} \Rightarrow \boxed{u(r) = \frac{\ln(b/r)}{\ln(b/a)}}$$

$$N = \iint u(r)\, r\,dr\,d\theta = \pi\left(\frac{b^2 - a^2}{2 \ln b/a} - a^2\right)$$

Integrated radial current:

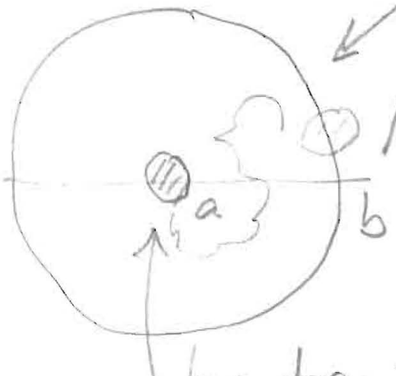
$$J = \int r J \, d\theta = \frac{2\pi D}{\ln b/a}$$

Time: $\boxed{T = \frac{N}{J} = \frac{1}{2D}\left(\frac{b^2 - a^2}{2} - a^2 \ln b/a\right) \approx \frac{b^2}{4D}}$

Can macrophage find bacteria in alveoli by random motion ?

Application: MΦ vs. bacteria

alveole: $b = 300\ \mu m$



Estimate of $D \approx \frac{1}{4}\frac{\delta x^2}{\delta t} \approx 11 \mu m^2/min$

$\delta t = 5$ min

$\delta x \approx 3 \mu m$

bacteria: $a = 20\ \mu m$

$$T \approx \frac{b^2}{4D} \approx \frac{300^2}{44 \cdot 60 \, min/hr} \approx 34\ hr$$

Bacter. replic. = 5 hr

Need $\frac{34}{5} \approx 7$ mφ to clear

Chemotaxis

$u(x,t)$ – bact. density

$c(x,t)$ – released chem. density

Chemotactic advection: $J_{ch} = u \underbrace{\alpha \nabla c}_{\text{driving veloc.}}$

$$\begin{cases} u_t = \nabla\cdot\Big[\underbrace{D_u \nabla u}_{J_D} - \underbrace{u\,\gamma \nabla c}_{J_{ch}}\Big] + \underset{\uparrow\ \text{source (replic./surv.)}}{\boxed{f(u)}} \\ c_t = \nabla\cdot\Big(D_c \nabla c\Big) + \boxed{g(u,c)} \end{cases}$$

Typical c-source: $g = \underbrace{\alpha u}_{\text{production}} - \underbrace{\beta c}_{\text{decay}}$

Coupled Linear or NL

reaction-<u>diffusion</u>