Population genetics

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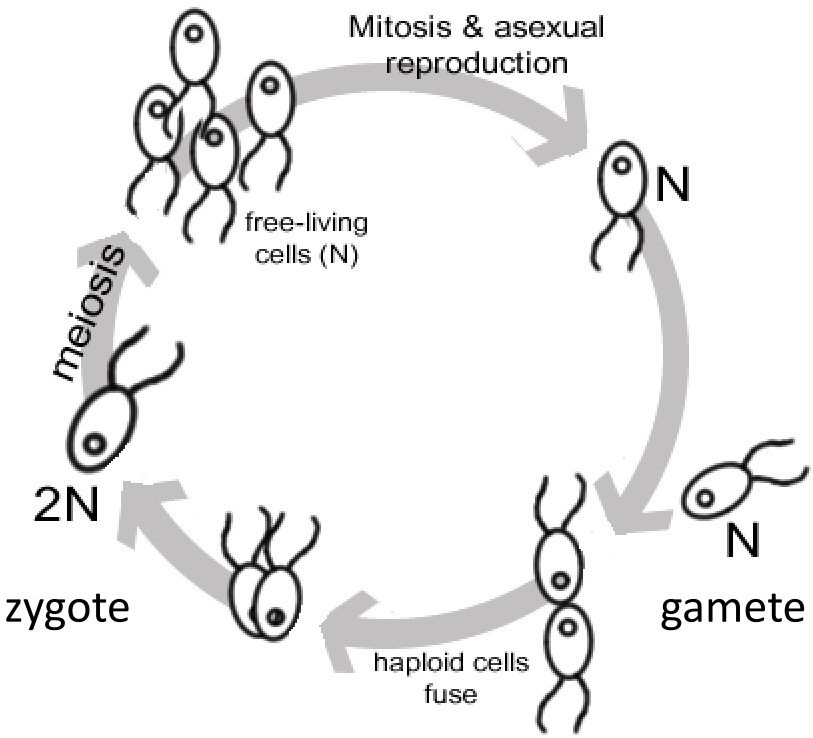
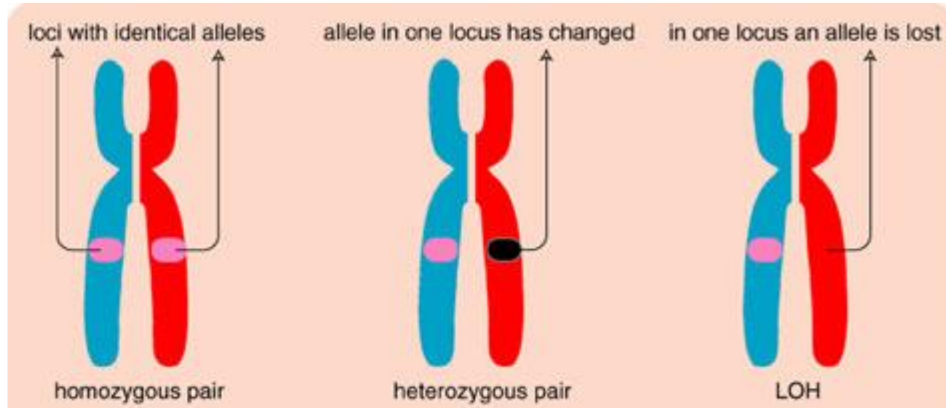
Topics

- Genotype & phenotype
- Reproduction: clonal and sexual (gene recombination)
- Fitness & dominance
- Adaptation and evolution (mutability vs. selection)
- Basic model: Fisher-Haldane-Wright, et al

Sexual reproduction cycle

http://en.wikipedia.org/wiki/Sexual_reproduction





Schematic rendition of a pair of homologous chromosomes under conditions of homozygosity, heterozygosity and LOH. Homologous chromosomes are those pair during mitosis, have the same morphology, and contain genes governing the same characteristics. Note that one of the chromosomes, say the blue one, came from the father, and the other, the red, from the mother.

Mendelian genetics: Hardy-Weinberg equilibrium

- Homogeneous population
- No stochastic effects
- No selection
- No mutation
- No migration, fragmentation et al

Diploid genotype with allele frequencies (gameto-pool): $p+q+\ldots=1$

A	B	C …
p	q	r …

2-allele zygote (HW) frequencies (Punnett table):

AA	AB	BB
x	y	z
p^2	2p q	q^2

1) Random mating

	AA	AB	BB
AA	x^2	xy	xz
AB	yx	y^2	yz
BB	zx	zy	z^2

2) Random (meiotic) recombination

AA+AA → AA (1)

AA+AB → AA (1/2), AB (1/2)

AA+BB → AB (1)

AB+AB → AA (1/4), AB (1/2), BB (1/4)

AB+BB → AB (1/2), BB (1/2)

BB+BB → BB (1)

Mating/recombination map

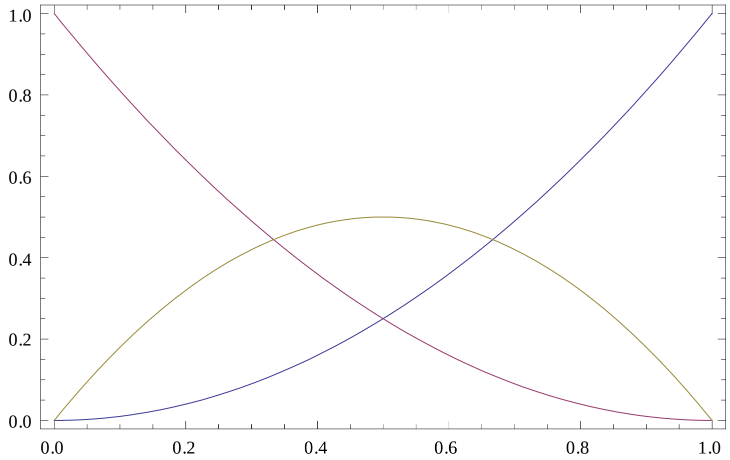
$$x_{n+1} = x^2 + xy + \frac{1}{4}y^2 = \left(x + y/2\right)^2$$

$$y_{n+1} = 2\left[\frac{1}{2}xy + \frac{1}{2}zy + xz\right] + \frac{1}{2}y^2 = 2\left(x + y/2\right)\left(z + y/2\right)$$

$$z_{n+1} = z^2 + yz + \frac{1}{4}y^2 = \left(z + y/2\right)^2$$

→ Equilibrium solution = H-W

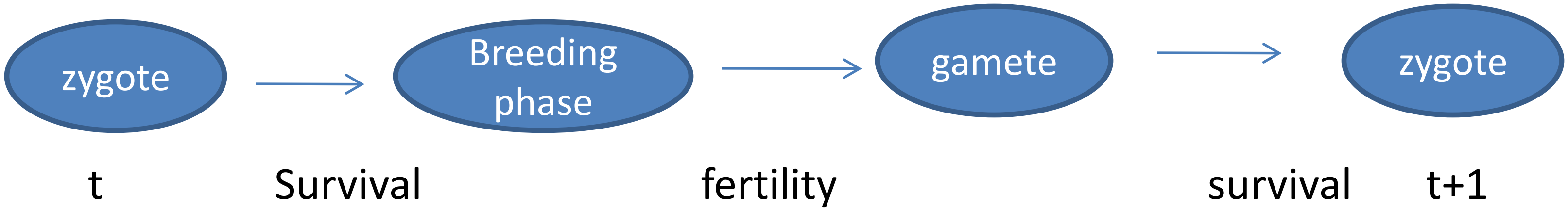
Single "generation map": (x,y,z)-> (x',y',z') attains H-W equilibrium



H-W Frequency distribution over 0<p<1

Reproductive fitness

Reproduction cycle



Genotype

AA	AB	BB
x	y	z

Allele pool

A	B
p=x+y/2	q=z+y/2

Random mating

AA	AB	BB
p^2	2p q	q^2

Survival fitness: $w_x : w_y : w_z$

Fisher-Haldane-Wright (FHW) map

$$p_{n+1} = \frac{\left(w_x p_n + w_y q_n\right) p_n}{w_x {p_n}^2 + 2 w_y p_n q_n + w_z {q_n}^2} = f\left(p_n\right)$$

Call

$$w_p = \left(w_x p + w_y q\right) \quad - \quad \text{"A-fitness"}$$

$$w_q = \left(w_z q + w_y p\right) \quad - \quad \text{"B-fitness"}$$

$$\bar{w} = w_x p^2 + 2 w_y p q + w_z q^2 \quad - \quad \text{"total fitness"}$$

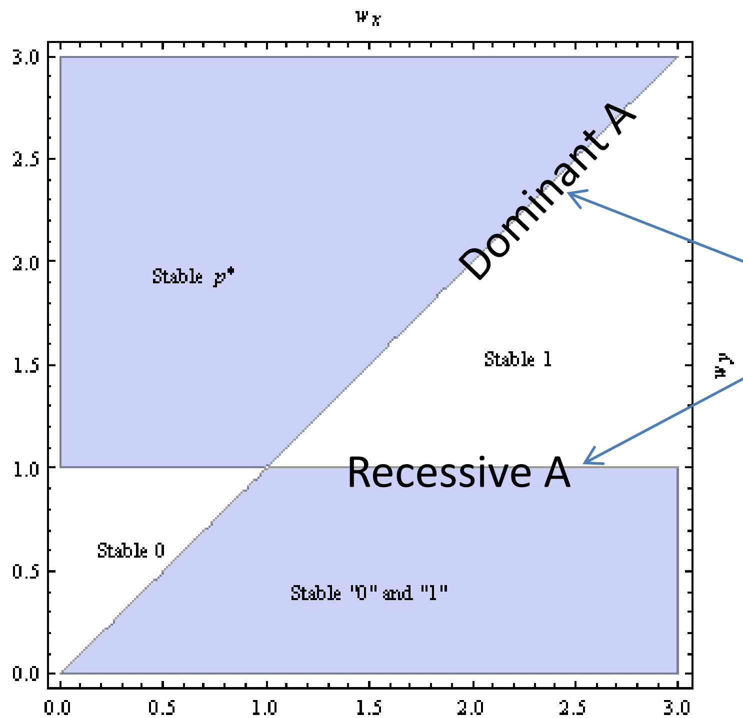
FHW- map: $$p' = \frac{w_p p}{\bar{w}} = f\left(p, u, v\right), \text{ for } \left(w_x, w_y, w_z\right) = \left(u, v, 1\right)$$

Analysis of FHW

Map: *q:=1-p;*

f[p,u,v]:=p (u p+v q)/(u p²+2 v p q+q²)

Equilibrium p^*	0	1	$\frac{1-v}{1+u-2v}$
Slope $f'(p^*)$	v	$\frac{v}{u}$	$-\frac{u(-2+v)+v}{u-v^2}$



Neutrally stable: f'(p)=1

Special cases

1. Selection for **dominant** allele A: u =v = 1+s > w = 1

   (AA & AB – better fit than BB)

$$p' = p\left[1 + \frac{sq^2}{1 + s\left(p^2 + 2pq\right)}\right] \to p^* = 1$$

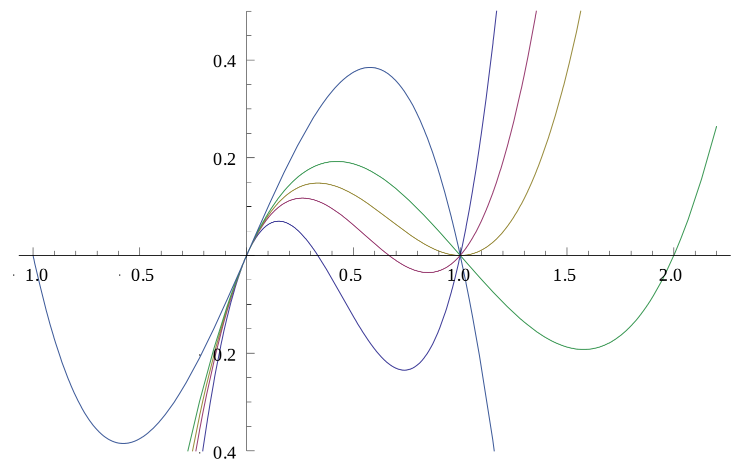
2. Selection for **recessive** allele A: u =1+s > v=w=1

$$p' = p\left[1 + \frac{spq}{1 + sp^2}\right] \to p^* = 1$$

Continuous DE model (s<<1) with relative fitness “gain- loss” h=a/b (modified “logistic”)

$$w_x \approx 1 + sa; w_y \approx 1 + sb; h = \frac{a}{b};$$

$$\boxed{\frac{dp}{dt} = sbp(1-p)\left[1 + (h-2)p\right]}$$



h-range= {-1;.5;1;1.5;3}
After h=1, fixation of A- allele (stable p=1)