Host-parasitoid

D. Gurarie

A discrete time (generation) model for host population *h*, and parasitoid *p* is given by a coupled system

$$\begin{aligned} h_{n+1} &= r f h_n \\ p_{n+1} &= c(1-f)h_n \end{aligned} \tag{1}$$

where replication rates $r > 1$ (for host) and $c > 0$ (for parasitoid), and $0 < f(h,p) < 1$- surviving (unparasitised) host fraction.

Nicholson-Bailey (NB) assume survival $f = e^{-ap}$, with *a* = "search intensity" arise from "random search" assumption (or "mass-action" law for parasite invasion) over time period *T*

$$\frac{dh}{dt} = -\alpha hp \Rightarrow h(T) = h_0 e^{-\alpha T p}; a = \alpha T \tag{2}$$

More generally, a function of *p* alone, $f(p)$ should decrease with p. Equilibrium state of (1) is computed via

$$\begin{aligned} f(p^*) &= 1/r; \\ h^* &= \frac{p^*}{c(1-1/r)}; \end{aligned}$$

and its Jacobian matrix

$$J = \begin{bmatrix} 1 & -rA/c \\ c(1-1/r) & A \end{bmatrix}; \text{ with } A = \frac{-p^* f'(p^*)}{1-1/r} > 0 \tag{3}$$

Matrix *J* has $\det = rA;\ \mathrm{tr} = 1 + A$. Applying Jury stability ($|\lambda_j| \le 1$) condition: $|\det| \le 1; |\mathrm{tr}| < 1 + \det$, we get

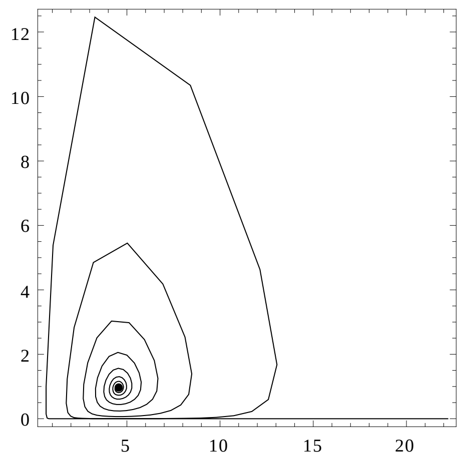
$$\begin{aligned} \det &= rA \le 1 \\ tr &= 1 + A \le 1 + rA \end{aligned}$$

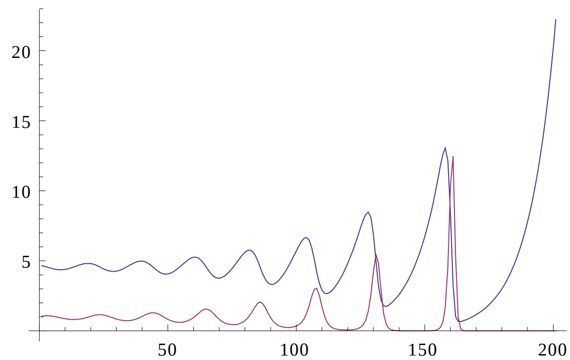
The 2nd (trace) holds automatically as *r>1*, the 1st (det) imposes some constraints on function *f* and *r* by (3).

Exercise: Study 3 cases below, including Nicholson-Bailey:

$f(p)$	A	det	Stability
1) e^{-ap}	$\frac{\ln r}{r-1}$	$\frac{r\ln r}{r-1}>1$ (all r>1)	unstable
2) $\mathrm{P}\log\left(e^{p-1}\right)$			?
3) $\frac{1}{1+p^m}$			(depends on m)?

Instability of equilibrium doesn't tell what happens to solutions at large time. N-B model exhibits unphysical behavior - divergent solutions $h_n, p_n \to \infty$





One drawback of NB is fixed "survival rate/parasite" = e^{-a}, independent of host numbers. An improved version (#2 in the Table) replaces "mass-action" predation pattern of (2) by "satiated predation" $\dot{h}=-\alpha(h)p$, with $\alpha=\frac{\alpha_0 h}{b+h}$, b = "half-level" of H. It has implicit solution

$$b\ln\left(H(t)/H_0\right)+H(t)-H_0=-\alpha_0 tP$$

which product-log survival fraction

$$f(H_0,P)=\frac{H(T)}{H_0}=\frac{b}{H_0}\mathrm{Plog}\left(\frac{H_0}{b}e^{-aP+H_0/b}\right);a=\frac{\alpha_0 T}{b} \quad (4)$$

-still unstable.

Exercise: Examine numerically cases (2-3) in the table at large time.