Logistic (LG) and modified logistic (MLG) growth

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Per capita growth rate G(y) is density dependent for several reasons: (i) resource limitations for large y, (ii) mating limitations for 'small' y

LG: $G(y) = a(1 - y/N)$; a= max(GR), N - CC

MLG: ...

Derivation of LG and MLG

I. Resource depletion (1.5 in Britton)

Premises: (i) resource $R = R_0 - \gamma y$ (= 'total' - 'occupied by y'); (ii) GR $G = \alpha R - \beta$. Derive LG from (i)-(ii), and compute *a* and *N* (1.5)

Natural examples:

1) habitat invasion (meta-population): R - available, y - invaded

2) spread of infectious disease SI: S -susceptible (resource); I - infected population (=y); P=S+I - total (fixed). Colonization (invasion/infection) => $G(S) = \beta \cdot S - r$, β - transmission rate (per I-host), r - recovery rate => LG: $\frac{dI}{dt} = (\beta S - r) I = a\left(1 - \frac{I}{K}\right) I$. Compute {a,K}, show a>0 if $P > r/\beta$, or BRN $R_0 = \frac{\beta P}{r} > 1$.

II. Resource allocation and intra-species competition

Resource allocation variable: $n = R/y$ (= mean 'R/host'). Premise: threshold n_0 determines 'replication/growth' or 'death'

$$n < n_0 \text{ - death}$$
$$n > n_0 \text{ - growth}$$

Possible cases:

1) Winner takes all

Divide $y = w + (y - w)$ "winners" + "losers" . Winners ($w = \min(R/n_0; y)$) take n_0 - resource (each one), and leave nothing to + "losers". Proliferation/death depend on allocated resource

	w	(y-w)
p.c. resource	n_0	0
Proliferation rate	b	0
Death rate	d	d

Get DE for y(t)

$$\dot{y} = b \min(y, R/n_0) - dy \tag{1.1}$$

Problem: (i) compute and plot RGR ($= \dot{y}/y$): $G(n/n_0)$ and $G(y)$;

(ii) show (1.1) has stable equilibrium $y^* = (\ldots) R/n_0$, compute it.

(iii) Solve DE (1.1) with $y(0) = y_0$ and plot solutions.

Hint: multicase solutions:

$$y_0 < R/n_0 \Rightarrow y(t) = \begin{cases} \ldots; t < t_0 = \ldots \\ \ldots; t > t_0 \end{cases}$$

$$y_0 > R/n_0 \Rightarrow y(t) = \ldots$$

2) Equal allocation: n=R/y

Assume n-dependent sigmoid RGR:

$$G(n/n_0) = a \frac{(n/n_0)^p - 1}{a/d + (n/n_0)^p}$$; p>1 - Hill exponent

Get MLG

$$\dot{y} = a \left[\frac{1 - (y/N)^p}{1 + \frac{d}{a}(y/N)^p} \right] y \tag{1.2}$$

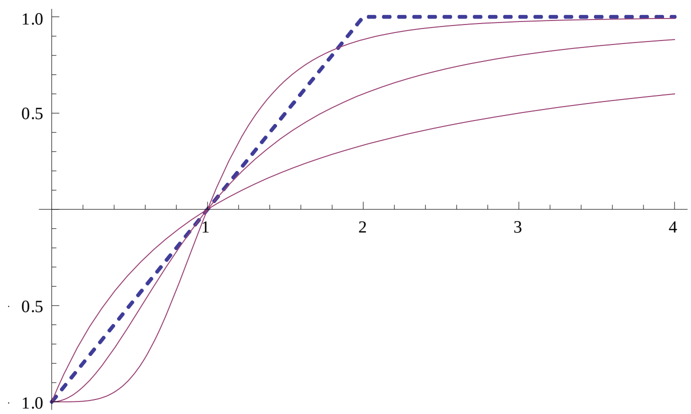

Figure 1: G(y) for (1.1) and (1.2) (p=1,2,4)

III. Mating probability

Has $G(y) = a(1 - y/N)y$, - increases for small y (depensatory), decreases for large y. MLG: $\dot{y} = a(1 - y/N)y^2$.

Dynamic resource (VL-type system)

Coupled ODE:

$$\begin{cases} \dot{R} = \alpha(R) - \delta(R)y \\ \dot{y} = G(R)y - d \cdot y \end{cases} \tag{1.3}$$

Quasi-equilibrium ("fast equilibrating" resource) $y \approx \dfrac{\alpha(R)}{\delta(R)}$, solve to get $R(y)$. Reduced y-dynamics (MGL)

$$\dot{y} = G(R(y))y - d \cdot y$$