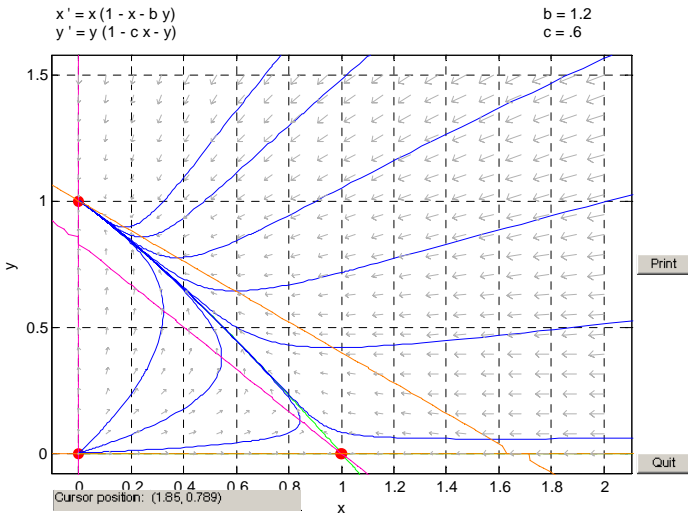
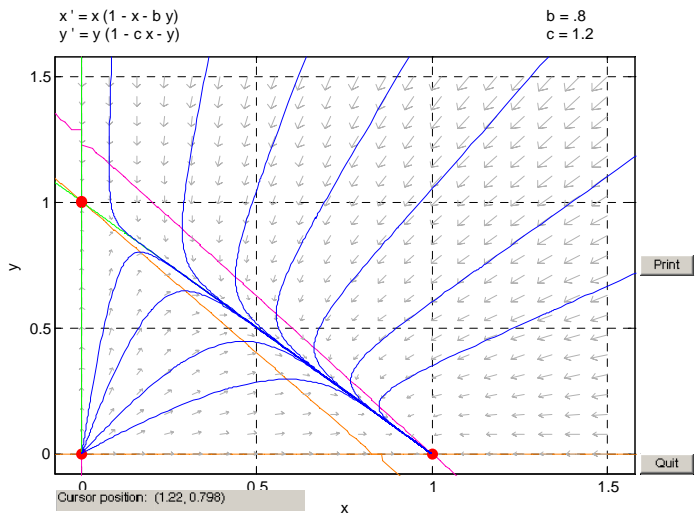


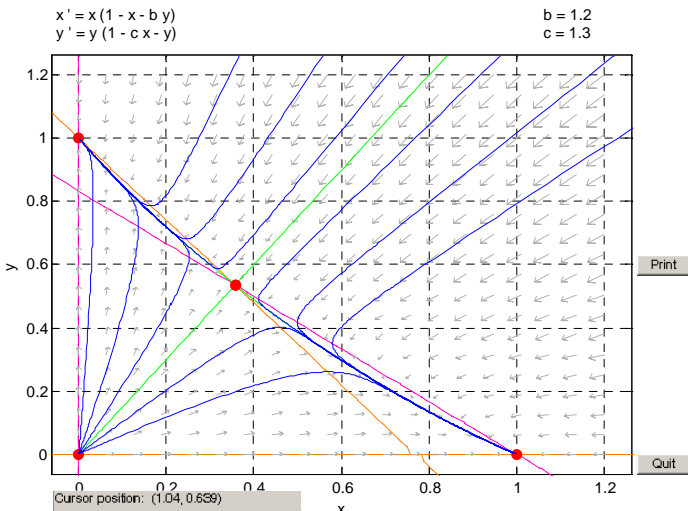
4 competition outcomes



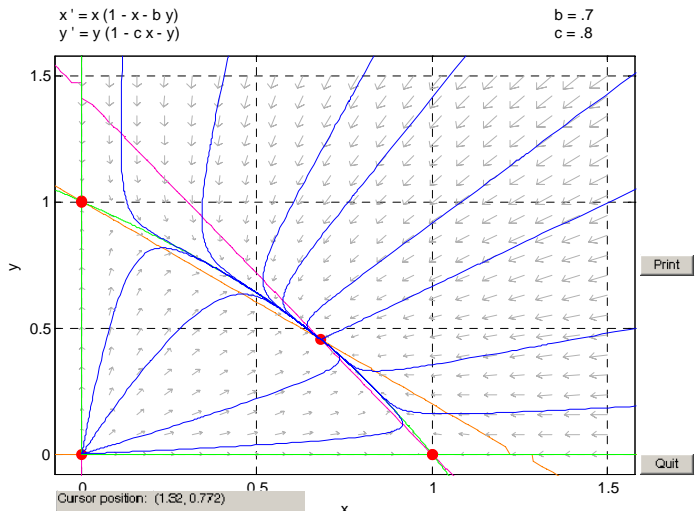
The forward orbit from (0.74, 1.5) --> a possible eq. pt. near (1.4e-013, 1).
 The backward orbit from (0.74, 1.5) left the computation window.
 Ready.
 Computing the field elements.
 Ready.



The backward orbit from (0.51, 1.1) left the computation window.
 Ready.
 The forward orbit from (0.91, 0.91) --> a possible eq. pt. near (1, -2.6e-012).
 The backward orbit from (0.91, 0.91) left the computation window.
 Ready.



The backward orbit from (0.25, 0.93) left the computation window.
 Ready.
 The forward orbit from (0.54, 0.69) --> a possible eq. pt. near (1, -4.4e-014).
 The backward orbit from (0.54, 0.69) left the computation window.
 Ready.



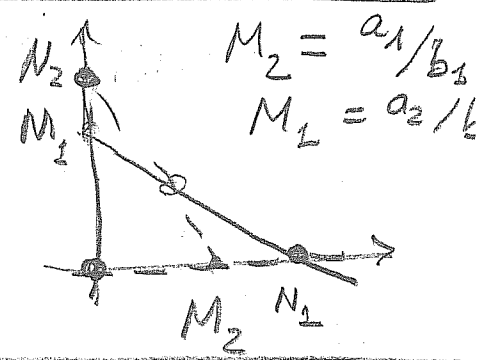
The backward orbit from (0.77, 1.4) left the computation window.
 Ready.
 The forward orbit from (1.3, 1.2) --> a possible eq. pt. near (0.68, 0.45).
 The backward orbit from (1.3, 1.2) left the computation window.
 Ready.

Analysis of VL competition

$$\begin{cases} \dot{x} = a_1 \left(1 - \frac{x}{N_1}\right) x - b_1 x y \\ \dot{y} = a_2 \left(1 - \frac{y}{N_2}\right) y - b_2 x y \end{cases} \quad \begin{array}{l} a_{1,2} - \text{max growth rates} \\ N_{1,2} - \text{c.e.} \\ b_{1,2} - \text{inhibition coefficient} \end{array}$$

Intersept form:

$$\begin{cases} a_1 \left(1 - \frac{x}{N_1} - \frac{y}{M_1}\right) x \\ a_2 \left(1 - \frac{x}{M_2} - \frac{y}{N_2}\right) y \end{cases}$$



Jacobian

$$J = \begin{bmatrix} a_1 \left(1 - \frac{2x}{N_1} - \frac{y}{M_1}\right) - b_1 x & -b_1 x \\ -b_2 y & a_2 \left(1 - \frac{x}{M_2} - \frac{2y}{N_2}\right) \end{bmatrix}$$

Equil:

$(0, 0)$	$(N_1, 0)$	$(0, N_2)$	(x^*, y^*) - coexis
$\begin{bmatrix} a_1 & 0 \\ 0 & a_2 \end{bmatrix}$	$\begin{bmatrix} -a_1 & -b_1 N_1 \\ 0 & a_2 \left(1 - \frac{N_1}{M_2}\right) \end{bmatrix}$	$\begin{bmatrix} a_1 \left(1 - \frac{N_2}{M_1}\right) & 0 \\ -b_2 N_2 & -a_2 \end{bmatrix}$	$\begin{bmatrix} & \\ & \end{bmatrix}$

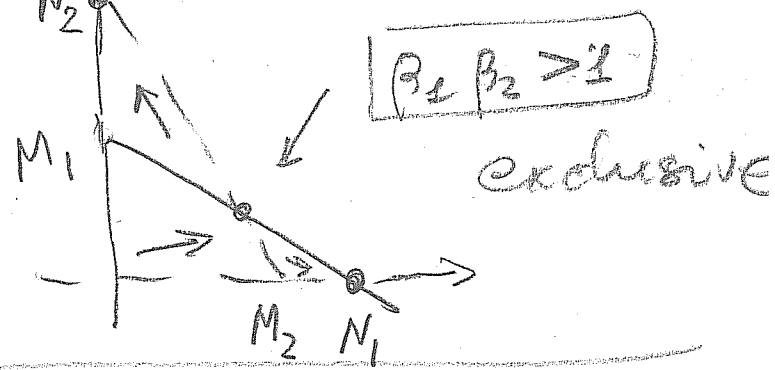
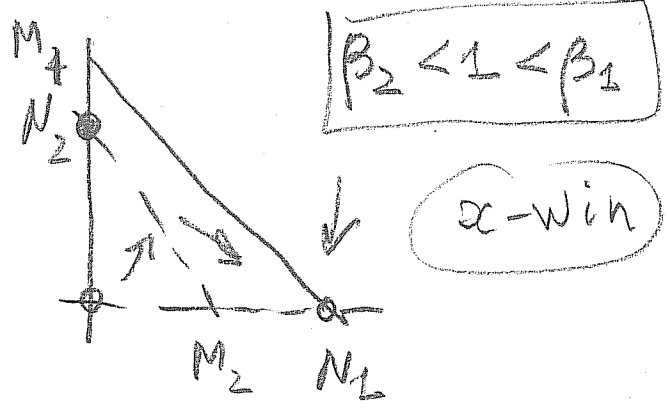
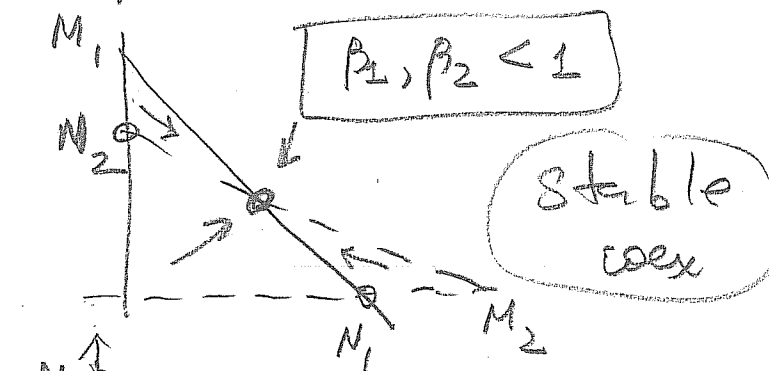
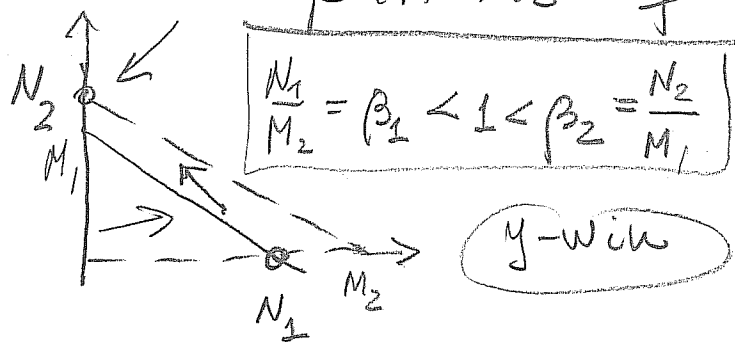
Analysis: essential parameters for outcome

$$\beta_2 = \frac{N_2}{M_1} = \frac{b_1 N_2}{a_1} - \text{"y-fitness" (suppression of x)}$$

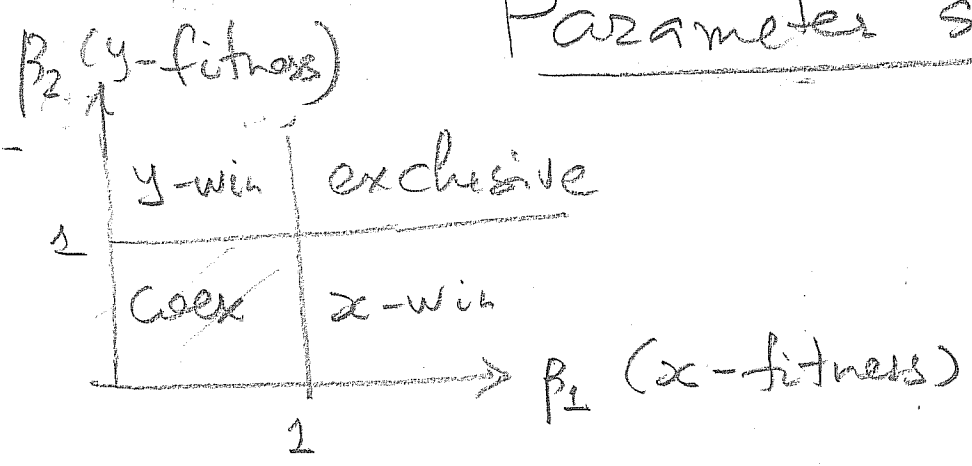
$$\beta_1 = \frac{N_1}{M_2} = \frac{b_2 N_1}{a_2} - \text{"x-fitness" (— " — y)}$$

4 patterns of competition

(2)



Parameter space



Problem: (i) Fill in table and determine pattern for a system

$$\begin{cases} \dot{x} = x(1 - \frac{x}{3}) - a_1xy \\ \dot{y} = 2(1 - \frac{y}{N_2}) - b_2xy \end{cases}$$

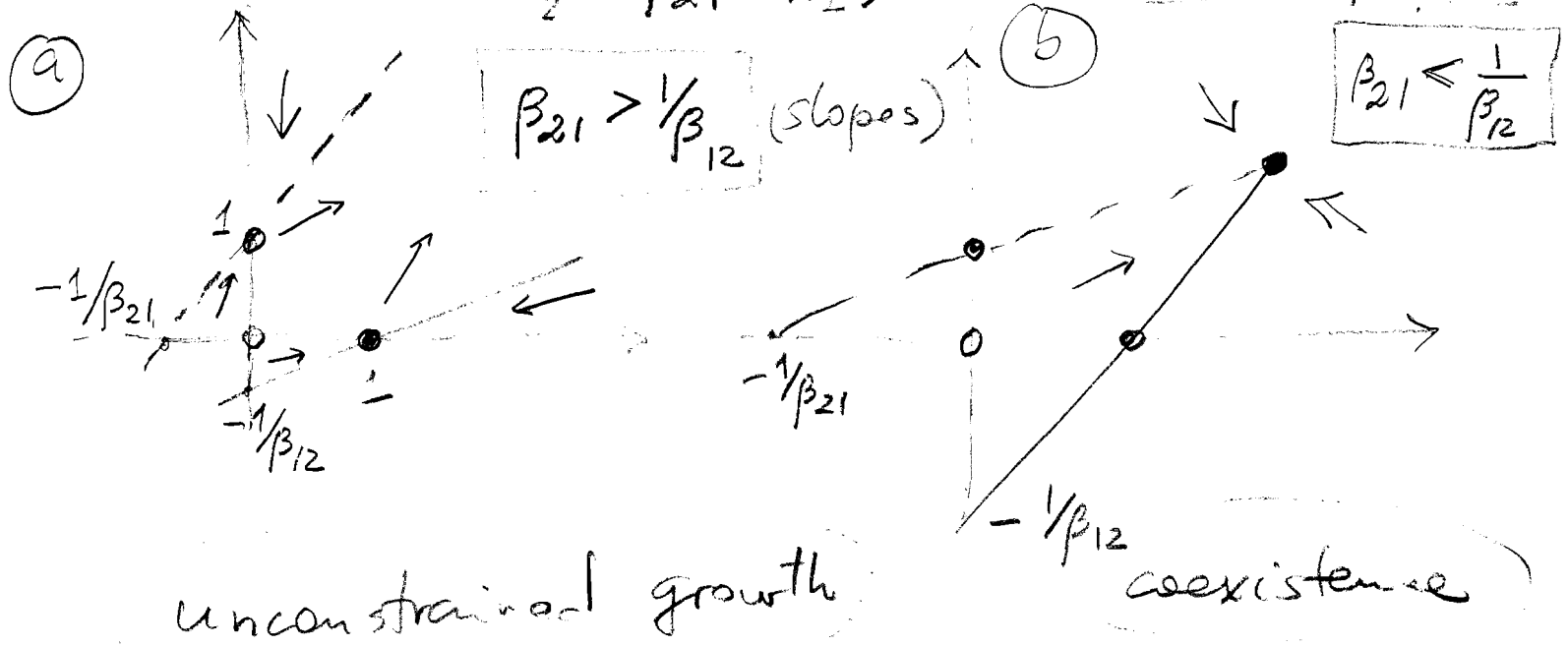
N_2	b_2	β_1	β_2	Outcome
1	1			
0.4	0.5			
2	0.5			

(ii) Verify your analysis by phase-plots in 3 cases

Mutualism & symbiosis

(5)

(3)
$$\begin{cases} \dot{x} = r_1 x \left(1 - \frac{x}{N_1} + \beta_{12} \frac{y}{N_2}\right) \\ \dot{y} = r_2 y \left(1 - \frac{y}{N_2} + \beta_{21} \frac{x}{N_1}\right) \end{cases} \xrightarrow{\text{Rescale}} \begin{cases} \dot{X} = X(1 - X + \beta_{12} Y) \\ \dot{Y} = \rho Y(1 - Y + \beta_{21} X) \end{cases}$$



Jacobian:
$$A = \begin{bmatrix} 1 - 2X + \beta_{12} Y & \beta_{12} X \\ \rho \beta_{21} Y & \rho(1 - 2Y - \beta_{21} X) \end{bmatrix}$$

	$(1, 0)$	$(0, 1)$	(X^*, Y^*) - coexist
$0, 0$	$(1, 0)$	$(0, 1)$	(X^*, Y^*) - coexist
$\begin{bmatrix} 1 & 0 \\ 0 & \rho \end{bmatrix}$	$\begin{bmatrix} -1 & \beta_{12} \\ 0 & \rho(1 + \beta_{21}) \end{bmatrix}$	$\begin{bmatrix} 1 + \beta_{12} & 0 \\ \rho \beta_{21} & -\rho \end{bmatrix}$	$\begin{bmatrix} -X^* & \beta_{12} X^* \\ \rho \beta_{21} Y^* & -Y^* \end{bmatrix}$ $\text{tr} < 0$
source	saddle	saddle	$\text{det} = \rho X^* Y^* (1 - \beta_{12} \beta_{21})$
(a) $\beta_{21} \beta_{12} > 1$	— " —	— " —	unphysical
(b) $\beta_{21} \beta_{12} < 1$	— " —	— " —	sink