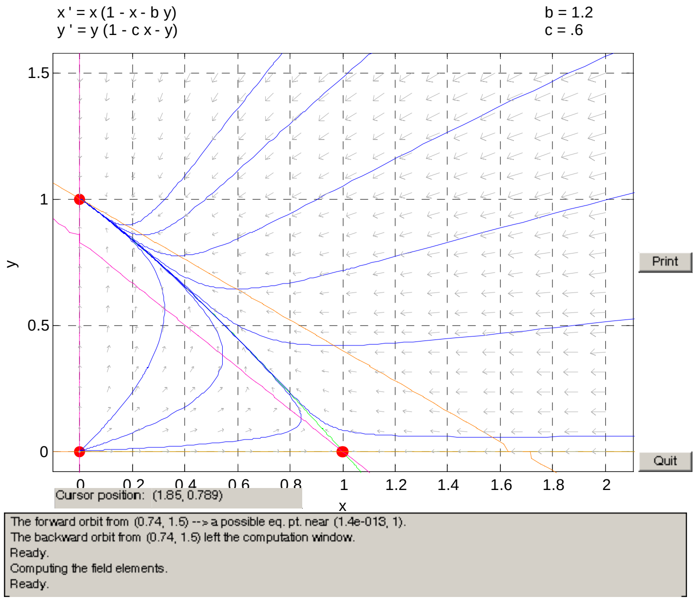
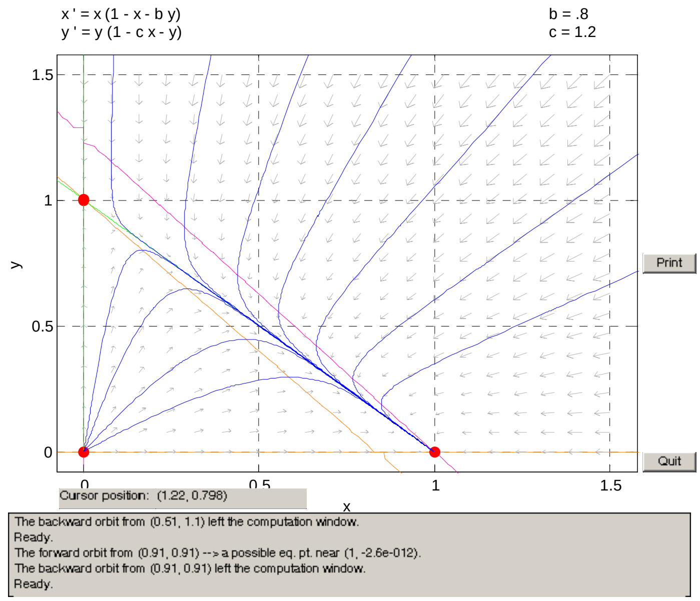
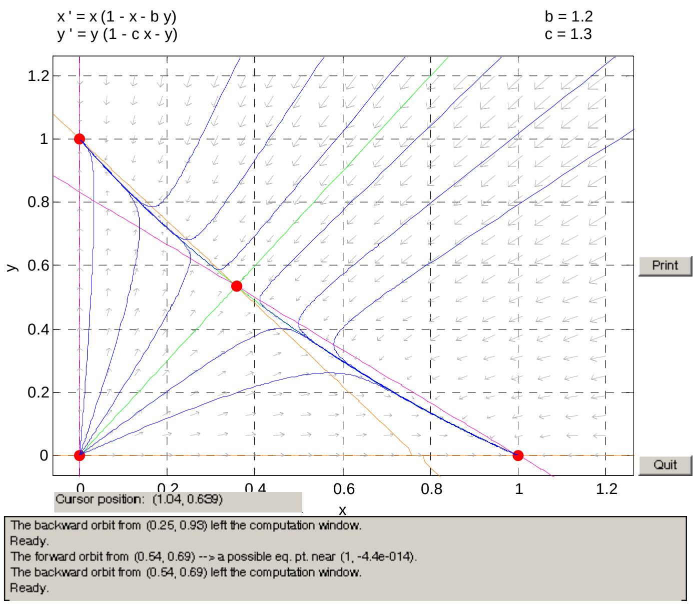
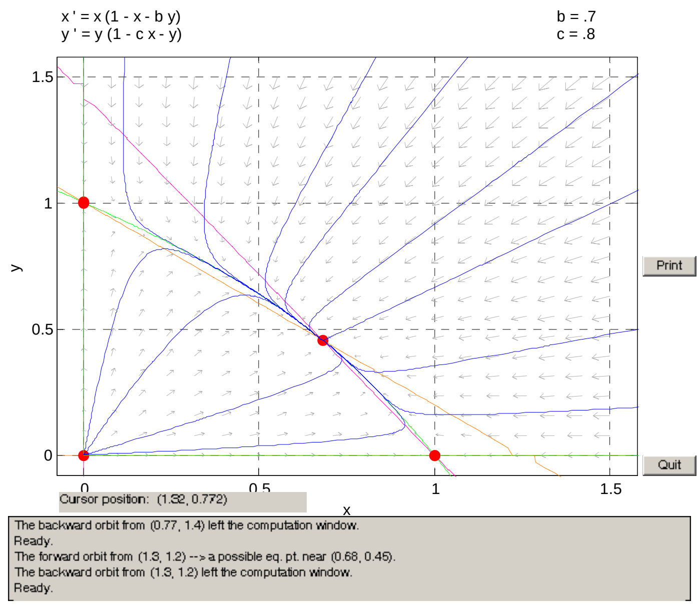
4 competition outcomes









D.GURAR

Analysis of VL competition

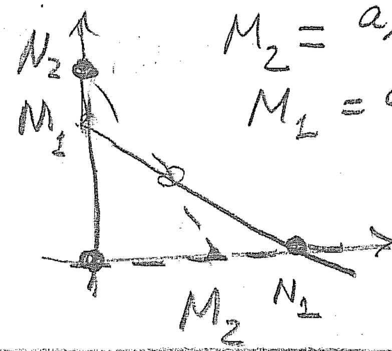
$$\begin{cases} \dot{x} = a_1\left(1-\frac{x}{N_1}\right)x - b_1 xy \\ \dot{y} = a_2\left(1-\frac{y}{N_2}\right)y - b_2 xy \end{cases}$$

$a_{1,2}$ – max growth rates
$N_{1,2}$ – c.c.
$b_{1,2}$ – inhibition coeffici

Intersept form:

$$\begin{cases} a_1\left(1 - x/N_1 - y/M_1\right)x \,; \\ a_2\left(1 - x/M_2 - y/N_2\right)y \,; \end{cases}$$

$M_2 = a_1/b_1$
$M_1 = a_2/b$



Jacobian

$$J = \begin{bmatrix} a_1\left(1-\frac{2x}{N_1}-y/M_1\right) & -b_1 x \\ -b_2 y & a_2\left(1-\frac{x}{M_2}-\frac{2y}{N_2}\right) \end{bmatrix}$$

Equil:

$(0,0)$	$(N_1\ 0)$	$(0,N_2)$	$(x^*\ y^*)$ - coexis
$\begin{bmatrix} a_1 & 0 \\ 0 & a_2 \end{bmatrix}$	$\begin{bmatrix} -a_1 & -b_1 N_1 \\ 0 & a_2\left(1-\underbrace{\boxed{\frac{N_1}{M_2}}}_{\beta_1}\right) \end{bmatrix}$	$\begin{bmatrix} a_1\left(1-\overbrace{\boxed{\frac{N_2}{M_1}}}^{\beta_2}\right) & 0 \\ -b_2 N_2 & -a_2 \end{bmatrix}$	$\begin{bmatrix} & \\ & \end{bmatrix}$

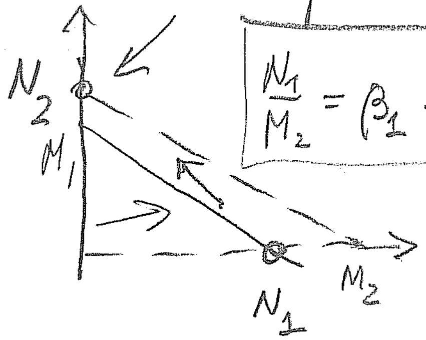
Analysis: essential parameters for outcome

$\beta_2 = \frac{N_2}{M_1} = \frac{b_1 N_2}{a_1}$ – "y-fitness" (suppression of x)

$\beta_1 = \frac{N_1}{M_2} = \frac{b_2 N_1}{a_2}$ – "x-fitness" (— " — y)

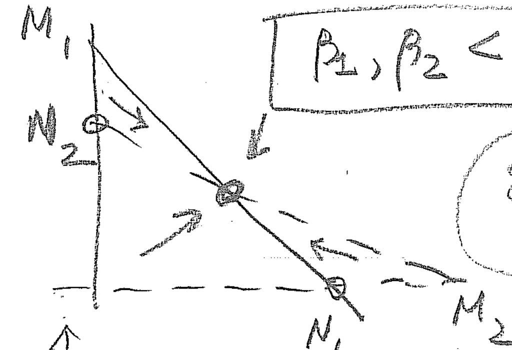
4 patterns of competition

$\boxed{\frac{N_1}{M_2} = \beta_1 < 1 < \beta_2 = \frac{N_2}{M_1}}$



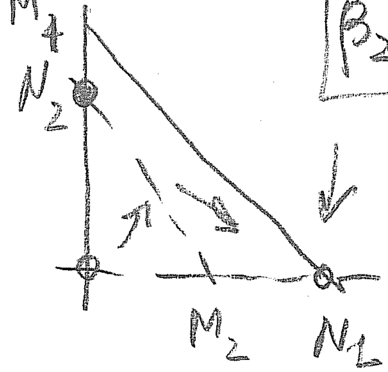
y-win

$\boxed{\beta_1, \beta_2 < 1}$



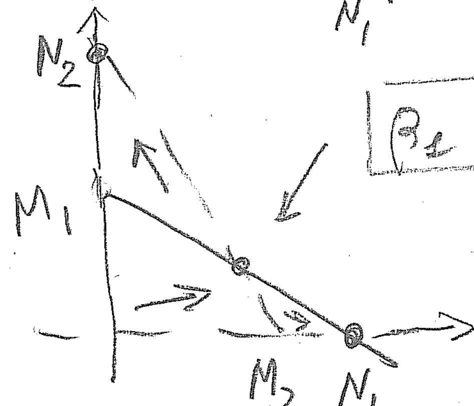
Stable coex

$\boxed{\beta_2 < 1 < \beta_1}$



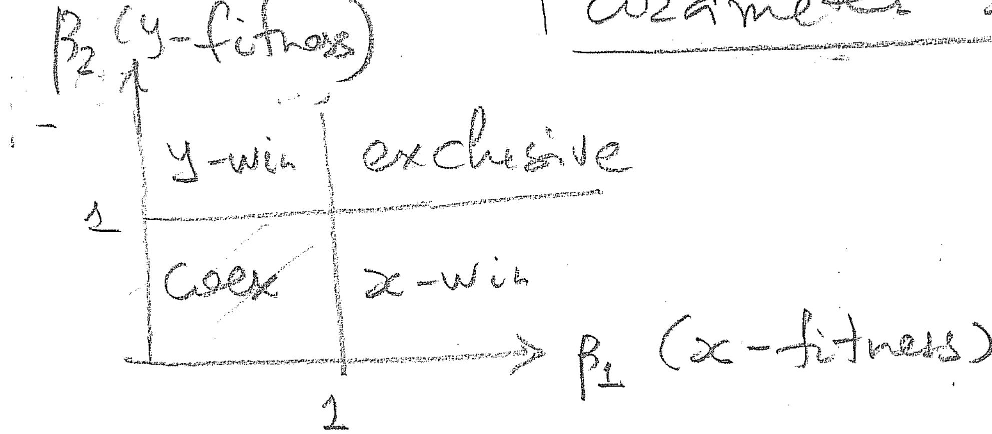
x-win

$\boxed{\beta_1 \beta_2 > 1}$



exclusive

Parameter space



Problem: (i) Fill in table and determine pattern for a system
$$\begin{cases} \dot{x} = x\left(1-\frac{x}{3}\right) - 2xy \\ \dot{y} = 2\left(1-\frac{y}{N_2}\right) - b_2 xy \end{cases}$$

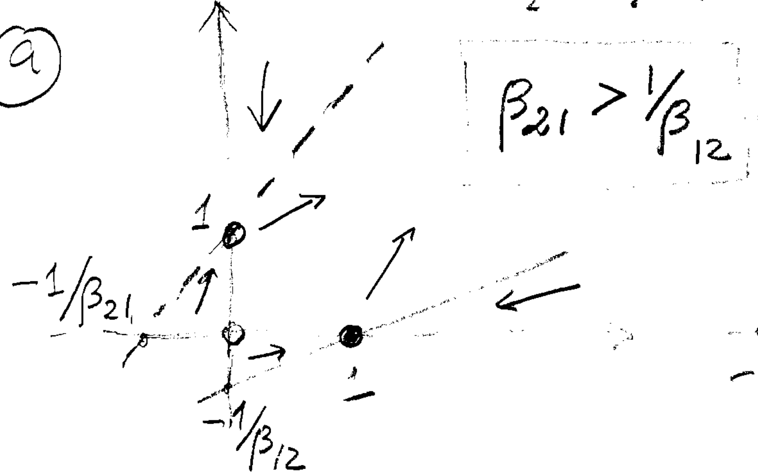
N_2	b_2	β_1	β_2	Outcome
1	1			
.4	.5			
2	.5			

(ii) Verify your analyze by phase-plots in 3 cases

Muticalism & symbiosis

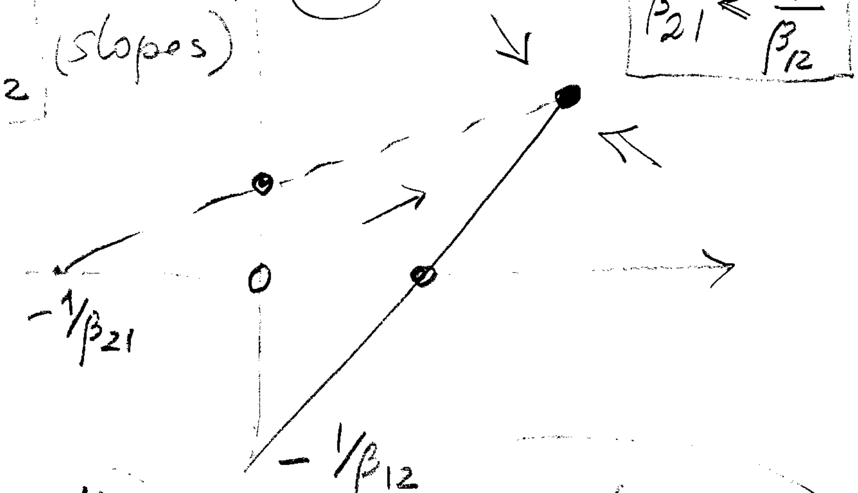
$$(3)\begin{cases} \dot{x} = r_1 x\left(1 - \frac{x}{N_1} + \beta_{12}\, y/N_2\right) \\ \dot{y} = r_2 y\left(1 - y/N_2 + \beta_{21}\, x/N_1\right) \end{cases} \xrightarrow{\text{Rescale}} \begin{cases} \dot{X} = X(1 - X + \beta_{12} Y) \\ \dot{Y} = \rho Y(1 - Y + \beta_{21} X) \end{cases}$$

(a) $\boxed{\beta_{21} > 1/\beta_{12}}$ (slopes)



unconstrained growth

(b) $\boxed{\beta_{21} < \frac{1}{\beta_{12}}}$



coexistence

Jacobian: $A = \begin{bmatrix} 1 - 2X + \beta_{12} Y & \beta_{12} X \\ \rho\beta_{21} Y & \rho(1 - 2Y - \beta_{21} X) \end{bmatrix}$

0,0	(1, 0)	(0, 1)	(X^*, Y^*) - coexist
$\begin{bmatrix} 1 & 0 \\ 0 & \rho \end{bmatrix}$	$\begin{bmatrix} -1 & \beta_{12} \\ 0 & \rho(1+\beta_{21}) \end{bmatrix}$	$\begin{bmatrix} (1+\beta_{12}) & 0 \\ \rho\beta_{21} & -\rho \end{bmatrix}$	$\begin{bmatrix} -X^* & \beta_{12} X^* \\ \rho\beta_{21} Y^* & -Y^* \end{bmatrix}$ tr<0 $\det = \rho X^* Y^*(1 - \beta_{12}\beta_{21})$
source	saddle	saddle	
(a) $\beta_{21}\beta_{12} > 1$ — " —	— " —	— " —	unphysical
(b) $\beta_{21}\beta_{12} < 1$ — " —	— " —	— " —	sink