Math 423 Homework 4

1. Let $\{f_n\}$ be a sequence of measurable functions on a measurable space $(X, \mathcal{M})$. Show that $\{x : \lim_n f_n(x) \text{ exists}\} \in \mathcal{M}$.

2. Let $(X, \mathcal{M})$ be a measurable space and f a function on X. Show that if $X = A \cup B$ with $A, B \in \mathcal{M}$, then f is measurable if and only if it is measurable on A and B.

3. Show that the supremum of an uncountable family of measurable functions need not be measurable.