Math 122
Test 2 - Review 1

1 Differential Equations

1.1 Separable

Solve the following differential equations:

1. $y' = 3(y+7)x^2$

2. $y' = \dfrac{x}{3y^2}$

3. $y' = \dfrac{4-2x}{3y^2-5}; \qquad y(1) = 3$

4. $y' = 2y - 3; \qquad y(0) = 2$

5. $y' = y + 1; \qquad y(0) = 1$

6. $y' = 3x^2y^2; \qquad y(0) = 1$

1.2 Slope Fields

Sketch the slope field and a likely solution curve for:

7. $\dfrac{dy}{dx} = y - x - 1 \qquad y(0) = 1$

8. $\dfrac{dy}{dx} = x^2 + y^2 \qquad y(0) = 1$

9. $\dfrac{dy}{dx} = 2x - 3y \qquad y(0) = 1$

1.3 Euler's Method

Use Euler's method with $h = 0.1$ to approximate $y(1)$ for the initial value problem:

10. $\dfrac{dy}{dx} = y - x - 1 \qquad y(0) = 1$

11. $\dfrac{dy}{dx} = x^2 + y^2 \qquad y(0) = 1$

12. $\dfrac{dy}{dx} = 2x - 3y \qquad y(0) = 1$

1.4 Linear

Solve the following differential equations:

13. $y' + (\cot x)y = 5e^{\cos x}$

14. $x^3y' + (2 - 3x^2)y = x^3$

15. $y' + \dfrac{3}{x}y = \dfrac{\sin x}{x^3}$

16. $xy' + 2y = 4x^2$

17. $x(\ln x)y' + y = 2\ln x$

18. $y' + (\cot x)y = 3\sin x \cos x$

19. $y' + 2y = 2x$

20. $y' + \dfrac{1}{x}y = \sin x$

21. $x\,y' = x - y \qquad y(2) = 1$

22. $y' - \frac{2y}{x+1} = (x^2 + 2x + 1)^2 \qquad y(0) = 1$

1.5 Applications

23. A tank contains 20 kg of salt dissolved in 5000 L of water. Brine that contains 0.03 kg of salt per liter of water enters the tank at a rate of 25 L/min. The solution is kept thorughly mixed and drains from the tank at the same rate. How much salt remains in the tank after half an hour?

24. A tank initially contains 100 gal of a solution that holds 40 lb of a chemical. A solution containing 2 lb/gal of the chemical runs into the tank at the rate of 2 gal/min and the mixture runs out at the rate of 2 gal/min. How much chemical is in the tank after 50 min?

25. A tank initially contains 100 gal of a solution that holds 30 lb of a chemical. Water runs into the tank at the rate of 2 gal/min and the solution runs out at the same rate. How much of the chemical remains in the tank after 20 min?

26. You are making High-Sea fruit juice. The bottle says that it contains at least 2% real fruit juice. Unfortunately, one of your employees made a vat of 500 gallons of juice that is 20% real fruit juice. You decide that you will pump water into the tank and bottle the over flow (the tank only holds 500 gallons). If the water is pumped in at 20 gallons per minute, how long can you continue to bottle the juice (it must be 2% real fruit juice.)

2 Parametric / Polar / Conic Sections

2.1 Parametric Equations

1. Sketch and identify the curve $x = 2 + \sin t$ and $y = 3 - \cos t$ for $0 \leq t \leq 2\pi$ by eliminating the parameter, and label the direction of increasing.

2. Find the arc length of the curve $x = t^2 + 2, y = t^3 - 3$ for $0 \leq t \leq 1$.

3. Find $\frac{dy}{dx}$ at the point where $t = \pi/4$ without eliminating t for $x = 5 - 2\cos t, y = 3 + \sin t$.

4. Sketch the curve $x = 2\cos t, y = 3\sin t$ for $0 \leq t \leq \pi/4$ by eliminating the parameter t and label the direction of increasing t.

5. Find $\frac{d^2y}{dx^2}$ at the point where $t = 3\pi/4$ without eliminating t for $x = 5 - \cos t, y = 3 + \sin t$.

6. Find the equation of the tangent line to $x = t^2, y = 3t^3$ at the point $(4, 24)$

7. For the curve given by $x = t^2 + t - 2$ and $y = t^3 - 2t - 1$, find the slope and concavity at the point $(4, 3)$.

2.2 Polar Equations

8. Express the polar equation $r = 4\sec\theta\tan\theta$ in rectangular form.

9. Express $x(x^2 + y^2) = 2(3x^2 - y^2)$ in polar form.

Graph the following polar equations:

10. $r = 3\sin 3\theta$

11. $r = 1 - 2\sin\theta$

12. $r = \theta$

13. $r = 1 - \sin\theta.$

14. $r^2 = 1 + \sin\theta.$

15. $r = 2 + 2\cos\theta.$

Find the polar equations for each of the equations given in rectangular form.

16. $x = 2$

17. $xy = 4$

18. $x^2 - y^2 = 1$

19. $x^2 + (y-2)^2 = 4$

20. Find the area common to both $r = 1 - \cos\theta$ and $r = 1 + \cos\theta$

21. Find the are inside $r = \sin\theta$ and outside $r = 1 - \cos\theta$.

22. Find the area inside both $r = 3\cos\theta$ and $r = 1 + \cos\theta$.

23. Find the area of the region enclosed by $r = 2 + 2\cos\theta$.

24. Find the area of the region enclosed by the cardioid $r = 2(1 + \cos\theta)$

25. Find the area of the region that is inside the circle $r = 1$ and outside the cardioid $r = 1 - \cos\theta$

26. Find the area of the region enclosed by $r = \cos 2\theta$.

27. Find the area of the region that is common to $r = a(1 + \sin\theta)$ and $r = a(1 - \sin\theta)$.

28. Find the slope of the tangent to the curve $r = 3 - 2\sin\theta$ at the point $\theta = \pi$.

29. Find the slope of the tangent to the curve $r = 2\sin\theta$ at the point $\theta = \pi/3$.

2.3 Conic Sections

30. Find the equation of the parabola with vertex $(1, 3)$ and directrix $x = -3$.

31. Find the equation of the ellipse with a major axis length of 12, a minor axis length of 8, center at $(0, 0)$, and foci on the x-axis.

32. Find the equation of the ellipse with a major axis length of 16, a minor axis length of 4, center at $(1, 4)$, and foci on $y = 4$.

33. Find the equation of the hyperbola with foci $(2, -6)$ and $(2, 10)$ and vertices 6 units apart.

34. Determine the rotation angle that will eliminate the xy-term

$$27x^2 + 9\sqrt{3}xy + 18y^2 + 6x + 3y = 0$$

35. Determine the rotation angle that will eliminate the xy-term

$$4x^2 + xy + 4y^2 + 9x - 7y = 0$$

36. Determine the rotation angle that will eliminate the xy-term

$$5x^2 + 10\sqrt{3}xy + 15y^2 + 7x - 7y = 0$$

37. Find the eccentricity of the conic described by $r = \dfrac{2}{1 + 4\sin\theta}$.

38. The directrix of the conic described by $r = \dfrac{4}{1 + 4\cos\theta}$. is how many units to the right of the pole?

39. Identify the conic section represented by $r = \dfrac{2}{2 - \cos\theta}$

Answers

Section 1

1. $y = -7 + Ce^{x^3}$

2. $y = \sqrt[3]{\frac{x^2}{2} + C}$

3. $y^3 - 5y = 4x - x^2 + 9$

4. $y = \frac{1}{2}(e^{2x} + 3)$

5. $y = 2e^x - 1$

6. $y = \frac{1}{1 - x^3}$

10. $y(1) \approx 0.406$

11. $y(1) \approx 7.18$

12. $y(1) \approx 0.478$

13. $y = \frac{1}{\sin x}\left[-5e^{\cos x} + C\right]$

14. $y = \frac{x^3}{3} + Cx^3e^{1/x^2}$

15. $y = \frac{1}{x^3}\left[-\cos x + C\right]$

16. $y = x^2 + FracCx^2$

17. $y = \frac{1}{\ln x}\left[(\ln x)^2 + C\right]$

18. $y = \sin^2 x + \frac{C}{\sin x}$

19. $y = -\frac{1}{2} + x + Ce^{-2x}$

20. $y = \frac{1}{x}\left[-x\cos x + \sin x + C\right]$

21. $y = \frac{x}{2}$

22. $y = \frac{(x+1)^5 + 2(x+1)^3}{3}$

23. 38.11 kg.

24. 141.1 lbs.

25. 20.11 lbs.

26. 57 minutes

Section 2

2. $S = \frac{1}{27}(13\sqrt{13} - 8)$

3. $\frac{dy}{dx} = \frac{1}{2}$

5. $\frac{d^2y}{dx^2} = -2\sqrt{2}$

6. $y = 9x - 12$

7. 2, concave up.

8. $x^2 = 4y$

9. $r = 2(3cos\theta - \sin\theta\tan\theta)$

16. $r = 2\sec\theta$

17. $r^2\cos\theta\sin\theta = 4$

18. $r^2\cos^2\theta - r^2\sin^2\theta = 1$

19. $r = 4\sin\theta$

20. Area $= \frac{3\pi}{2} - 4$

21. Area $= 1 - \frac{\pi}{4}$

22. Area $= \frac{5\pi}{4}$

23. $A = 6\pi$

24. 6π

25. $2 - \frac{\pi}{4}$

26. $A = \frac{\pi}{2}$

27. $A = \frac{a^2}{3}(3\pi - 8)$

28. $m = \frac{3}{2}$

29. $m = -\sqrt{3}$

30. $(y-3)^2 = 16(x-1)$

31. $\frac{x^2}{36} + \frac{y^2}{16} = 1$

32. $\frac{(x-1)^2}{64} + \frac{(y-4)^2}{4} = 1$

33. $\frac{(y-2)^2}{9} - \frac{(x-2)^2}{55} = 1$

34. $\frac{\pi}{6}$

35. $\frac{\pi}{4}$

36. $-\frac{\pi}{6}$

37. 4

38. 1

39. Ellipse