

Math 122
Quiz 8 Review

Find the n -th degree Taylor polynomial centered at $x = a$ and $R_n(x)$ for:

1. $f(x) = \arctan x$ $a = 0$ $n = 2$

2. $f(x) = \cos x$ $a = \frac{\pi}{4}$ $n = 3$

3. $f(x) = \frac{1}{\sqrt{1-x}}$ $a = 0$ $n = 4$

Write out the first four non-zero terms for the Taylor Series for:

4. $f(x) = e^x$ centered at $a = 1$

5. $f(x) = \sin x$ centered at $a = \pi$

6. $f(x) = \cos 2x$ centered at $a = \pi$

7. $f(x) = \sqrt{x}$ centered at $a = 1$

8. $f(x) = (1+x)^{1/2}$ centered at $a = 0$

9. $f(x) = \ln x$ centered at $a = 1$

Find the Maclaurin series for:

10. $f(x) = \sin 2x$

11. $f(x) = x \sin x$

12. $f(x) = xe^x$

13. $f(x) = \cos \sqrt{x}$

14. $f(x) = e^{-x^2}$

15. $f(x) = \frac{1 - \cos x}{x}$

Determine if the following series converge or diverge:

16. $\sum_{n=2}^{\infty} \frac{(\ln n)^2}{n}$

17. $\sum_{n=1}^{\infty} \frac{1}{n^2} \cos\left(\frac{1}{n}\right)$

18. $\sum_{n=1}^{\infty} \frac{1}{n^{-2}}$

19. $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n}}$

Answers

1. $P_2(x) = x, R_2(x) = \frac{6z^2 - 2}{3!(1+z^2)^3}x^3$
2. $P_3(x) = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right) - \frac{\sqrt{2}}{2!}\frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right)^2 + \frac{\sqrt{2}}{3!}\frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right)^3, R_3(x) = \frac{1}{4!}\left(x - \frac{\pi}{4}\right)^4 \cos z$
3. $P_4(x) = 1 + \frac{1}{2}x + \frac{3}{8}x^2 + \frac{5}{16}x^3 + \frac{35}{128}x^4, R_4(x) = \frac{945}{2^5 5! (1-z)^{11/2}}x^5$
4. $e^x = e + e(x-1) + \frac{e}{2}(x-1)^2 + \frac{e}{3!}(x-1)^3 + \dots$
5. $\sin x = -(x-\pi) + \frac{1}{6}(x-\pi)^3 - \frac{1}{5!}(x-\pi)^5 + \frac{1}{7!}(x-\pi)^7 - \dots$
6. $\cos 2x = 1 - 2(x-\pi)^2 + \frac{2}{3}(x-\pi)^4 - \frac{4}{45}(x-\pi)^6 + \dots$
7. $\sqrt{x} = 1 + \frac{(x-1)}{2} - \frac{1}{8}(x-1)^2 + \frac{1}{16}(x-1)^3 - \dots$
8. $(1+x)^{1/2} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16} - \dots$
9. $\ln x = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \frac{1}{4}(x-1)^4 + \dots$
10. $2x - \frac{8x^3}{3!} + \frac{32x^5}{5!} + \dots$
11. $x^2 - \frac{x^4}{3!} + \frac{x^6}{5!} - \dots$
12. $x + x^2 + \frac{x^3}{2} + \frac{x^4}{3!} + \dots$
13. $1 - \frac{x}{2} + \frac{x^2}{4!} - \frac{x^3}{3!} + \dots$
14. $1 - x^2 + \frac{x^4}{2!} - \frac{x^6}{3!} + \dots$
15. $\frac{x}{2} - \frac{x^3}{4!} + \frac{x^5}{6!} - \frac{x^7}{8!} + \dots$
16. Diverges
17. Converges
18. Diverges
19. Diverges