

Math 122
Quiz 5 Review

Find the general solution to the following differential equations:

1. $\frac{dy}{dx} = \frac{x}{y^2}$

2. $x(y-1)y' = y$

3. $\frac{dy}{dx} - 4xy = x$

4. $2y' + 3y = e^{-x}$

5. $\frac{dy}{dx} - \frac{2x}{x^2+1}y = x$

6. $\frac{dy}{dx} + y \cot x = 1$

7. $\cos x \frac{dy}{dx} + y \sin x = 0$

8. $\frac{dy}{dx} - 3y = e^{3x} \sin x$

9. $y^2 y' = 3x^2$

10. $y' = x^3 y^2 + y^2$

11. $y' = 5 - 2y$

12. $\frac{dy}{dx} + 2xy = 4x$

13. $x \ln x \frac{dy}{dx} + (y - \ln x) = 0$

Solve the following initial value problems:

14. $y' = -4xy^2, \quad y(0) = 1$

16. $xy' + y = \ln x, \quad y(1) = 2$

15. $y' - y = \frac{e^x}{x}, \quad y(e) = 0$

17. $e^y y' = 2x + 1, \quad y(0) = 1$

18. $y' = \frac{xe^x}{y}, \quad y(0) = -5$

19. Find a curve in the xy -plane that passes through $(0, 3)$ and whose tangent line at a point (x, y) has slope $\frac{2x}{y^2}$

20. At time $t = 0$, a tank contains 4 lb of salt dissolved in 100 gal of water. Suppose that brine containing 2 lb of salt per gallon of water is allowed to enter the tank at a rate of 5 gal/min and the mixed solution is drained from the tank at the same rate. Find the amount of salt in the tank after 10 min.

21. A tank initially contains 60 gallons of water containing 40 pounds of orange drink mix. A drink mix solution containing 1 pound of mix per gallon is added to the tank at the rate of 2 gallons per minute, and the solution in the tank is drained off at the rate of 3 gallons per minute. How much drink mix is in the tank after 30 minutes?

22. The coffee urn in Tomlinson holds 25 quarts when it is full. In the process of cleaning it, Joe has filled the urn with soapy water with a 2% concentration of soap. Now he is rinsing it out by running pure water into the urn at 2.5 quarts per minute and letting the soapy mixture run out of the urn at the same rate. Let $S(t)$ denote the number of quarts of soap in the urn after t minutes.

a. Set up a differential equation for S .

b. How long will it take until the concentration of soap in the urn reaches 0.02%?

23. Suppose that a tank initially contains 50 gallons of pure water and brine containing b pounds of salt per gallon is pumped into the tank a rate of 5 gal/min and the well-stirred mixture leaves at the same rate.

a. Determine b so there are 5 pounds of salt in the tank after 1 hour.

b. Determine b so there are 10 pounds of salt in the tank after 1 hour.

Answers

1. $y = \left(\frac{3}{2}x^2 + C\right)^{1/3}$
2. $y - \ln|y| = \ln|x| + C$
3. $y = -\frac{1}{4} + Ce^{2x^2}$
4. $y = e^{-x} + Ce^{-3/2x}$
5. $y = (x^2 + 1) \left(\frac{1}{2} \ln|x^2 + 1| + C\right)$
6. $y = c \csc x - \cot x$
7. $y = c \cos x$
8. $y = e^{3x} (-\cos x + C)$
9. $y = \sqrt[3]{3x^3 + C}$
10. $y = \frac{4}{x^4 + 4x + C}$
11. $y = \frac{5}{2} + Ce^{-2x}$
12. $y = 2 + Ce^{-x^2}$
13. $y = \frac{\ln x}{2} + \frac{C}{\ln x}$
14. $y = \frac{1}{2x^2 + 1}$
15. $y = e^x (\ln x - 1)$
16. $y = \ln x - 1 + \frac{3}{x}$
17. $y = \ln(x^2 + x + e)$
18. $y = -\sqrt{2xe^x - 2e^x + 27}$
19. $y = (3x^2 + 27)^{1/3}$
20. 81.1 lb
21. 27.5 pounds
22. 46 minutes
23. a. $b = .1 \text{ lb/ gal}$
b. $b = .2 \text{ lb/ gal}$