Fractional Perimeters

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For a Borel set $E \subset \mathbb{R}^n$ and $0 < s < 1$, the fractional s-perimeter of E is given by

$$P_s(E) = \int_E \int_{E^c} \frac{1}{|x-y|^{n+s}}\, dx\, dy, \tag{1}$$

where E^c denotes the complement of E in $\mathbb{R}^n$ and $|\cdot|$ the Euclidean norm on $\mathbb{R}^n$. The functional P_s is an $(n-s)$-dimensional perimeter on Borel sets on $\mathbb{R}^n$, as $P_s(\lambda E) = \lambda^{n-s} P_s(E)$ for $\lambda > 0$. It is non-local in the sense that it is not determined by the behavior of E in a neighborhood of ∂E.

The limiting behavior of fractional s-perimeters as $s \to 1^-$ and $s \to 0^+$ turns out to be very interesting. Dávila, extending results by Bourgain, Brezis & Mironescu from 2002, shows that for a bounded Borel set $E \subset \mathbb{R}^n$ of finite perimeter,

$$\lim_{s \to 1^-} (1-s) P_s(E) = \alpha_n P(E),$$

where $P(E)$ is the perimeter of E and α_n is a constant depending on n.

Anisotropic fractional s-perimeters are natural generalizations of the Euclidean notions obtained by replacing the Euclidean norm $|\cdot|$ in (1) by an arbitrary norm $\|\cdot\|$. In the talk the limiting behavior of anisotropic s-perimeters as $s \to 1^-$ and $s \to 0^+$ is discussed. Applications to anisotropic fractional isoperimetric inequalities and anisotropic fractional Sobolev norms are also presented.