May 7, 2007

Syllabus in Dynamical Systems

- *Ordinary differential equations*
  Existence and uniqueness theorems; linear systems; critical points, periodic orbits, stability, bifurcations, hyperbolicity; linearization; stable, unstable, and center manifolds; Poincaré-Bendixsen theory.

- *Discrete dynamical systems*
  Periodic points: stability, hyperbolicity and bifurcations; linearization; stable manifold theory; invariant sets: nonwandering points, chain recurrence, attractors, hyperbolic sets; symbolic dynamics; one-dimensional dynamics, Sarkovskii's theorem; topological entropy.

References: (There is a lot of overlap in the coverage of the above topics in the following texts.)

1. R.C. Robinson, *Dynamical Systems: stability, symbolic dynamics, and chaos*
2. A. Katok and B. Hasselblatt, *Modern Theory of Dynamical Systems*
3. L. Perko, *Differential Equations and Dynamical Systems*, through 3.7
4. J. Cronin, *Differential Equations*, chapters 1-4 and parts of 5 and 6
5. J. Hale & H. Kocak, *Dynamics and Bifurcations*, parts I and III
6. J. Palis &W. de Melo, *Geometric Theory of Dynamical systems*, through 2.6
7. D. Ruelle, *Elements of Differentiable Dynamics and Bifurcation Theory*
8. R. Devaney, *An Introduction to Chaotic Dynamical Systems*, through 2.6
9. Z. Nitecki, *Differentiable Dynamics*, chapters 1, 2.0-2.2; 4.1-4.3; 5.1-5.2
10. M. Shub, *Global Stability of Dynamical Systems*, chapter 5
11. J. Guckenheimer & P. Holmes, *Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields*, chapters 1, 3 and 5
12. S. Wiggins, *Global Bifurcations and Chaos*, through 2.2b

The first two texts go far beyond the topics in the syllabus. Another book by Robinson, *An Introduction to Dynamical Systems, continuous and discrete* provides a lower-level introduction to some of these topics. The books by Ruelle, Hale & Kocak, Wiggins, and Guckenheimer & Holmes are to varying extents descriptive; where detailed proofs are missing in these texts, they can be found in the other texts. The book by Nitecki is somewhat dated (1971).

Note: The above should be construed as a sample syllabus. Should this exam be offered in the future, the selection of topics may be modified, for example depending on the coursework of a particular student, or to ensure the breadth and non-overlap requirements. In particular, for a student pursuing the ODE+PDE option in Differential Equations, the treatment of the differential equations topics listed above would be more detailed, and additional topics, such as Floquet theory and the method of averaging would likely be included. References for a student choosing this option would include items 1, 3, 4, 6, 11 from the list above, and the following:

13. J. Hale, *Ordinary Differential Equations*
14. P. Hartman, *Ordinary Differential Equations*
15. M. W. Hirsch, S. Smale, R. Devaney, *Differential Equations, Dynamical Systems, and an introduction to chaos*.
16. V. I. Arnold, *Ordinary Differential Equations*