May 7, 2007

Syllabus in Differential Equations

1. Special functions and series solutions:
   - Regular and singular points; Frobenius method;
   - Bessel (cylinder) functions (differential equations; series solutions; integral representations)
   - Orthogonal polynomials: Legendre/Jacoby; Hermite; Laguerre

   N. N. Lebedev, Special functions and their applications, (ch. 4, 5)

2. Partial differential equations and systems
   - 1-st order PDE: the method of characteristics (linear, quasi-linear, nonlinear).
   - 2-nd and higher order equations: classification; initial/boundary-value problem; well-posedness; stability; dissipation; dispersion.

3. Separation of variables and generalized Fourier-series expansions:
   - Sturm-Liuoville and multi-dimensional elliptic eigenvalue problems: orthogonality and completeness of eigenfunctions
   - Laplace, heat and wave equations in bounded regions.

4. Transform methods (Fourier; Laplace; Henkel); problems in unbounded regions.

5. Green's functions and fundamental solutions:
   - Green' identities;
   - Generalized functions
   - Fourier method: series and integral expansions of Green's functions
   - Symmetries and the Method of images: Green's functions for the Laplace, heat and wave equations in special regions (space; half-space; quadrant; slab; box; disk; ball; sphere)

6. Variational, Perturbation and Asymptotic methods:
   - Hamilton's principle of minimal action and applications: vibrating strings, membranes, etc. Energy conservation and causality.
   - The mini-max principle and Rayleigh-Ritz method for eigenfunctions of differential operators;
   - Regular perturbations (eigenvalue, boundary value perturbations)
   - Equations with large parameter: Stationary phase and Geometrical optics methods; Helmholtz equation.

References:

R. Courant, D. Hilbert, Methods of Mathematical Physics
G.F.D. Duff and D. Naylor, Differential Equations of Applied Mathematics, John Wiley & Sons
A. Nayfeh, Perturbation Methods
I. Stakgold, Green's functions and boundary-value problems, Wiley-Interscience
E. Zauderer, Partial Differential Equations of Applied Mathematics, Wiley-Interscience
(reference to most basic topics)
W. A. Strauss, Partial Differential Equations: An Introduction, Wiley
G. Whitham, Linear and nonlinear waves (ch. 1,2,3,5,7,11)

Note: The above syllabus is centered on Partial Differential Equations. The material is partly covered in MATH 445 and MATH 448. Some topics require additional reading. Should this exam be offered in the future, the selection of topics may be modified, for example depending on the coursework of a particular student, or to ensure the breadth and non-overlap requirements. One such specific option would cover a subset of the topics above (roughly corresponding to MATH 445) plus a selection of Ordinary Differential Equations topics, most of which are included in the Dynamical Systems syllabus. (That option cannot be chosen should the student attempt also Dynamical Systems.)