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Complex Analysis Syllabus

The syllabus for the qualifying exam in complex analysis may be divided into two main parts. Part I covers standard topics of a graduate course in complex analysis and corresponds roughly to MATH 425. Part II covers an agreed upon selection of topics building on Part I—see, e.g., Ahlfors, pp. 225–312; Hille, Vol. II; or Rudin, pp. 320–428.

1. Complex analysis. Analytic functions. Integration over paths in the complex plane. Index of a point with respect to a closed path. Cauchy's theorem and Cauchy's integral formula. Power series representation. Open mapping theorem. Singularities. Laurent expansion. Residue calculus. Maximum modulus theorem. Harmonic functions, mean value property, Poisson's formula. Conformal mappings, the Riemann mapping theorem. (This is largely the catalogue description of MATH 425.)

2. The syllabus for this part may be drawn from some of the following topics, to name a few: Dirichlet's problem; conformal mapping and conformal equivalence; analytic continuation and algebraic functions; Riemann surfaces; elliptic functions; discrete subgroups of the Möbius group; uniformization; metrics and curvature of complex domains; entire functions and Picard's Theorem; boundary behavior and spaces of analytic functions on the disc. (This is a sample list of topics. Should this exam be offered in the future, the selection of topics in this part may be modified, for example depending on the coursework of a particular student.)

Primary Reference for Part I :

J. B. Conway, Functions of one complex variable, 2nd Edition: Chapters 1-5; Chapter 6 Sections 1, 2; Chapter 7, Sections 1, 2, 4; Chapter 10, Sections 1, 2.

Additional references:

M. Ablowitz and A. Fokas, Complex variables, Cambridge Texts in Applied Mathematics.
L. V. Ahlfors, Complex Analysis, 3rd Edition
E. Hille, Analytic function theory, Volumes I, II, Ginn and Company
G. Jones and D. Singerman, Complex functions: an algebraic and geometric viewpoint, Cambridge Univ. Press.
S. Krantz, Complex analysis: the geometric viewpoint, MAA Carus Monograph.
Z. Nehari, Conformal mapping, Dover.
W. Rudin, Real and Complex Analysis, 3rd Edition