APPENDIX M: ROTATIONS ABOUT NONTRIVIAL AXES

Rotations due to r.f. pulses follow in the PO formalism the pattern of vector kinematics. In Section II.6 we have discussed rotations about the axes *Ox* and *Oy*. In order to treat the general case (rotation axis with any spatial orientation) we use spherical coordinates as shown in Figure M.1. The rotation axis makes an angle θ with *Oz* ($0 \le \theta \le 180^{\circ}$) and its horizontal projection makes the angle Φ with *Ox* (positive sense for Φ is from *Ox* toward *Oy*).

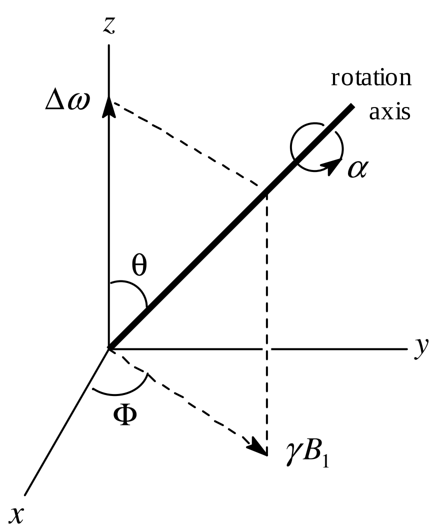

Figure M.1. Rotation axis in spherical coordinates.

If the r.f. pulse is applied on resonance, $\theta = 90^{\circ}$. For a deviation $\Delta\omega = \gamma B_o - \omega_{tr}$ from resonance, the angle θ is different from 90°; its value is defined by $\tan\theta = \gamma B_1/\Delta\omega$. The angle Φ is the transmitter phase which is always taken with respect to an initially established receiver phase.

The result of a rotation by an angle α (right hand rule) about the axis defined above is described in Table M.1.

Table M.1. Final Angular Momentum Components after Rotation by an Angle α about the Rotation Axis Defined by Angles θ and Φ.

Initial component	Final components		
I_x	$I_x(1-\cos\alpha)\times$ $\times\sin^2\theta\cos^2\Phi+$ $+I_x\cos\alpha$	$I_y(1-\cos\alpha)\times$ $\times\sin^2\theta\cos\Phi\sin\Phi+$ $+I_y\sin\alpha\cos\theta$	$I_z(1-\cos\alpha)\times$ $\times\sin\theta\cos\theta\cos\Phi-$ $-I_z\sin\alpha\sin\theta\sin\Phi$
I_y	$I_x(1-\cos\alpha)\times$ $\times\sin^2\theta\cos\Phi\sin\Phi-$ $-I_x\sin\alpha\cos\theta$	$I_y(1-\cos\alpha)\times$ $\times\sin^2\theta\sin^2\Phi+$ $+I_y\cos\alpha$	$I_z(1-\cos\alpha)\times$ $\times\sin\theta\cos\theta\sin\Phi+$ $+I_z\sin\alpha\sin\theta\cos\Phi$
I_z	$I_x(1-\cos\alpha)\times$ $\times\sin\theta\cos\theta\cos\Phi+$ $+I_x\sin\alpha\sin\theta\sin\Phi$	$I_y(1-\cos\alpha)\times$ $\times\sin\theta\cos\theta\sin\Phi-$ $-I_y\sin\alpha\sin\theta\cos\Phi$	$I_z(1-\cos\alpha)\times$ $\times\cos^2\theta+$ $+I_z\cos\alpha$

In Table M.2 we consider the particular case of an on-resonance pulse ($\theta = 90^o$).

Table M.2. Final Angular Momentum Components after Rotation by an Angle α about an Axis in the *xy* Plane (θ=90^o) Defined by Angle Φ.

Initial com-ponent	Final components		
I_x	$I_x \cos^2\Phi +$ $+I_x \cos\alpha \sin^2\Phi$	$I_y (1-\cos\alpha)\times$ $\times \cos\Phi \sin\Phi$	$-I_z \sin\alpha \sin\Phi$
I_y	$I_x (1-\cos\alpha)\times$ $\times \cos\Phi \sin\Phi$	$I_y \sin^2\Phi +$ $+I_y \cos\alpha \cos^2\Phi$	$I_z \sin\alpha \cos\Phi$
I_z	$I_x \sin\alpha \sin\Phi$	$-I_y \sin\alpha \cos\Phi$	$I_z \cos\alpha$

Very often the phase angle Φ can only take one of the "cardinal" values 0^o, 90^o, 180^o, 270^o. In this case, the product $\sin\Phi\cos\Phi$

vanishes and the expressions above become even simpler, as shown in Table M.3.

Table M.3. Final Angular Momentum Components after Rotation by an Angle α about an Axis in the *xy* Plane (θ=90º) with the Angle Φ a Multiple of 90º.

Initial component	Final components		
I_x	$I_x \cos^2 \Phi +$ $+I_x \cos\alpha \sin^2 \Phi$	0	$-I_z \sin\alpha \sin\Phi$
I_y	0	$I_y \sin^2 \Phi +$ $+I_y \cos\alpha \cos^2 \Phi$	$I_z \sin\alpha \cos\Phi$
I_z	$I_x \sin\alpha \sin\Phi$	$-I_y \sin\alpha \cos\Phi$	$I_z \cos\alpha$

Making Φ equal to 0º, 90º, 180º, 270º, in Table M.3, yields the rotation rules about *Ox*, *Oy*, $-Ox$, $-Oy$ (phase cycling). It is instructive to compare these results with those obtained by means of the vector representation.